2. Examine whether the following statements are true or false:

(i) $\{ a, b \} \not\subset \{ b, c, a \}$

(ii) $\{ a, e \} \subset \{ x : x$ is a vowel in the English alphabet$\}$

(iii) $\{ 1, 2, 3 \} \subset \{ 1, 3, 5 \}$

(iv) $\{ a \} \subset \{ a, b, c \}$

(v) $\{ a \} \in \{ a, b, c \}$

(vi) $\{ x : x$ is an even natural number less than $6\} \subset \{ x : x$ is a natural number which divides $36\}$

3. Let $A = \{ 1, 2, \{ 3, 4 \}, 5 \}$. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$ (ii) $\{3, 4\} \in A$ (iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$ (v) $1 \subset A$ (vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$ (viii) $\{1, 2, 3\} \subset A$ (ix) $\phi \in A$

(x) $\phi \subset A$ (xi) $\{\phi\} \subset A$

4. Write down all the subsets of the following sets

(i) $\{a\}$ (ii) $\{a, b\}$ (iii) $\{1, 2, 3\}$ (iv) ϕ

5. How many elements has P(A), if $A = \phi$?

6. Write the following as intervals :

(i) $\{x : x \in R, -4 < x \leq 6\}$ (ii) $\{x : x \in R, -12 < x < -10\}$

(iii) $\{x : x \in R, 0 \leq x < 7\}$ (iv) $\{x : x \in R, 3 \leq x \leq 4\}$

7. Write the following intervals in set-builder form :

(i) $(-3, 0)$ (ii) $[6 , 12]$ (iii) $(6, 12]$ (iv) $[-23, 5)$

8. What universal set(s) would you propose for each of the following :

(i) The set of right triangles. (ii) The set of isosceles triangles.

9. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C

(i) $\{0, 1, 2, 3, 4, 5, 6\}$

(ii) ϕ

(iii) $\{0,1,2,3,4,5,6,7,8,9,10\}$

(iv) $\{1,2,3,4,5,6,7,8\}$

1.9 Venn Diagrams

Most of the relationships between sets can be represented by means of diagrams which are known as *Venn diagrams*. Venn diagrams are named after the English logician, John Venn (1834-1883). These diagrams consist of rectangles and closed curves usually circles. The universal set is represented usually by a rectangle and its subsets by circles.

In Venn diagrams, the elements of the sets are written in their respective circles (Figs 1.2 and 1.3)

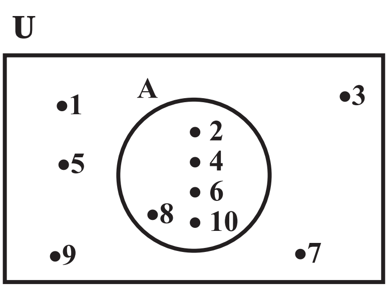


Fig 1.2

Illustration 1 In Fig 1.2, U = {1,2,3, ..., 10} is the universal set of which

A = {2,4,6,8,10} is a subset.

Illustration 2 In Fig 1.3, U = {1,2,3, ..., 10} is the universal set of which

A = {2,4,6,8,10} and B = {4, 6} are subsets, and also B ⊂ A.

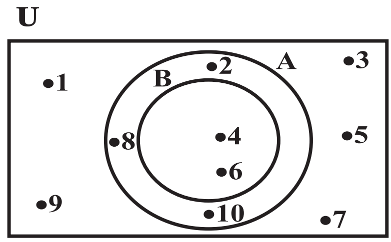


Fig 1.3

The reader will see an extensive use of the Venn diagrams when we discuss the union, intersection and difference of sets.

1.10 Operations on Sets

In earlier classes, we have learnt how to perform the operations of addition, subtraction, multiplication and division on numbers. Each one of these operations was performed on a pair of numbers to get another number. For example, when we perform the operation of addition on the pair of numbers 5 and 13, we get the number 18. Again, performing the operation of multiplication on the pair of numbers 5 and 13, we get 65. Similarly, there are some operations which when performed on two sets give rise to another set. We will now define certain operations on sets and examine their properties. Henceforth, we will refer all our sets as subsets of some universal set.

1.10.1 ***Union of sets*** Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once. The symbol '∪' is used to denote the *union*. *Symbolically, we write* A ∪ B *and usually read as 'A union B'*.

Example 12 Let A = { 2, 4, 6, 8} and B = { 6, 8, 10, 12}. Find A ∪ B.

Solution We have A ∪ B = { 2, 4, 6, 8, 10, 12}

Note that the common elements 6 and 8 have been taken only once while writing A ∪ B.

Example 13 Let A = { *a, e, i, o, u* } and B = { *a, i, u* }. Show that A ∪ B = A

Solution We have, A ∪ B = { *a, e, i, o, u* } = A.

This example illustrates that union of sets A and its subset B is the set A itself, i.e., if B ⊂ A, then A ∪ B = A.

Example 14 Let X = {Ram, Geeta, Akbar} be the set of students of Class XI, who are in school hockey team. Let Y = {Geeta, David, Ashok} be the set of students from Class XI who are in the school football team. Find X ∪ Y and interpret the set.

Solution We have, X ∪ Y = {Ram, Geeta, Akbar, David, Ashok}. This is the set of students from Class XI who are in the hockey team or the football team or both.

Thus, we can define the union of two sets as follows:

Definition 6 The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write. $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

The union of two sets can be represented by a Venn diagram as shown in Fig 1.4.

The shaded portion in Fig 1.4 represents $A \cup B$.

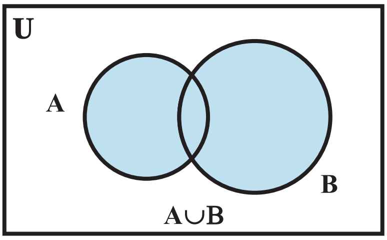


Fig 1.4

Some Properties of the Operation of Union

(i) $A \cup B = B \cup A$ (Commutative law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$
(Associative law)

(iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)

(iv) $A \cup A = A$ (Idempotent law)

(v) $U \cup A = U$ (Law of U)

1.10.2 ***Intersection of sets*** The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol '$\cap$' is used to denote the *intersection*. The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Example 15 Consider the sets A and B of Example 12. Find $A \cap B$.

Solution We see that 6, 8 are the only elements which are common to both A and B. Hence $A \cap B = \{ 6, 8 \}$.

Example 16 Consider the sets X and Y of Example 14. Find $X \cap Y$.

Solution We see that element 'Geeta' is the only element common to both. Hence, $X \cap Y = \{\text{Geeta}\}$.

Example 17 Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{ 2, 3, 5, 7 \}$. Find $A \cap B$ and hence show that $A \cap B = B$.

Solution We have $A \cap B = \{ 2, 3, 5, 7 \} = B$. We note that $B \subset A$ and that $A \cap B = B$.

Definition 7 The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

The shaded portion in Fig 1.5 indicates the intersection of A and B.

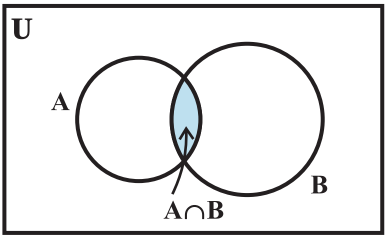


Fig 1.5

If A and B are two sets such that $A \cap B = \phi$, then A and B are called *disjoint sets.*

For example, let $A = \{ 2, 4, 6, 8 \}$ and $B = \{ 1, 3, 5, 7 \}$. Then A and B are disjoint sets, because there are no elements which are common to A and B. The disjoint sets can be represented by means of Venn diagram as shown in the Fig 1.6

In the above diagram, A and B are disjoint sets.

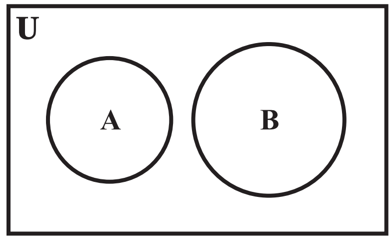


Fig 1.6

Some Properties of Operation of Intersection

(i) $A \cap B = B \cap A$ (Commutative law).

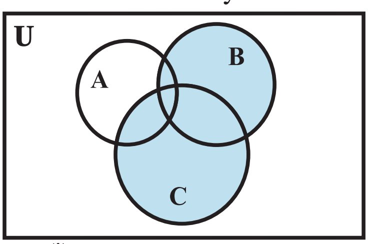
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U).

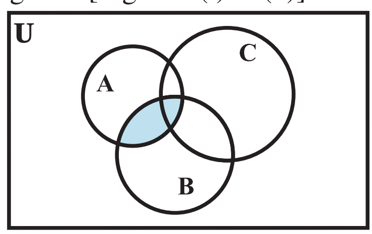
(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i. e., $\cap$ distributes over $\cup$

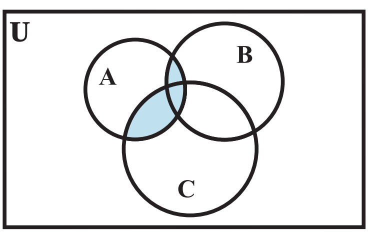
This can be seen easily from the following Venn diagrams [Figs 1.7 (i) to (v)].



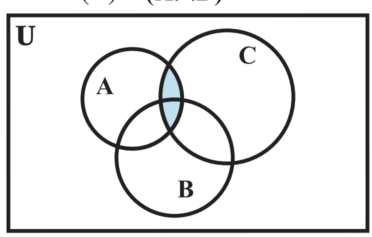
(i) **(B∪C)**



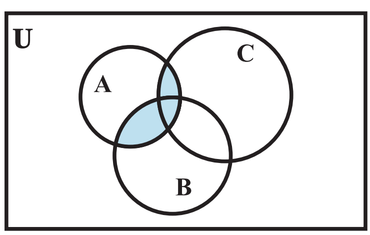
(iii) **(A∩B)**



(ii) **A∩(B∪C)**



(iv) **(A∩C)**



(v) **(A∩B)** ∪ (A∩C)

Figs 1.7 (i) to (v)

1.10.3 ***Difference of sets*** The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write A – B and read as " A minus B".

Example 18 Let A = { 1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8 }. Find A – B and B – A.

Solution We have, A – B = { 1, 3, 5 }, since the elements 1, 3, 5 belong to A but not to B and B – A = { 8 }, since the element 8 belongs to B and not to A.
We note that A – B ≠ B – A.

Example 19 Let V = { a, e, i, o, u } and B = { a, i, k, u}. Find V – B and B – V

Solution We have, V – B = { e, o }, since the elements e, o belong to V but not to B and B – V = { k }, since the element k belongs to B but not to V.

We note that V – B ≠ B – V. Using the set-builder notation, we can rewrite the definition of difference as

$$A - B = \{ x : x \in A \text{ and } x \notin B \}$$

The difference of two sets A and B can be represented by Venn diagram as shown in Fig 1.8.

The shaded portion represents the difference of the two sets A and B.

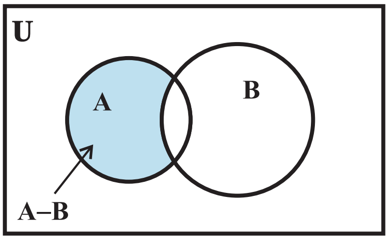


Fig 1.8

Remark The sets A – B, A ∩ B and B – A are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set as shown in Fig 1.9.

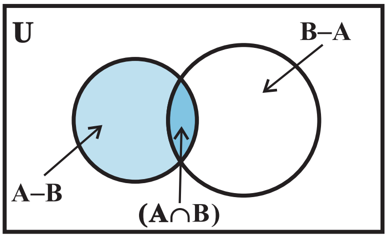


Fig 1.9

EXERCISE 1.4

1. Find the union of each of the following pairs of sets :
   (i) X = {1, 3, 5} Y = {1, 2, 3}
   (ii) A = [a, e, i, o, u} B = {a, b, c}
   (iii) A = {x : x is a natural number and multiple of 3}
   B = {x : x is a natural number less than 6}
   (iv) A = {x : x is a natural number and $1 < x \leq 6$ }
   B = {x : x is a natural number and $6 < x < 10$ }
   (v) A = {1, 2, 3}, B = ϕ
2. Let A = { a, b }, B = {a, b, c}. Is A ⊂ B ? What is A ∪ B ?
3. If A and B are two sets such that A ⊂ B, then what is A ∪ B ?
4. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8 }and D = { 7, 8, 9, 10 }; find

(i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$ (iv) $B \cup D$
(v) $A \cup B \cup C$ (vi) $A \cup B \cup D$ (vii) $B \cup C \cup D$

5. Find the intersection of each pair of sets of question 1 above.

6. If A = { 3, 5, 7, 9, 11 }, B = {7, 9, 11, 13}, C = {11, 13, 15}and D = {15, 17}; find
(i) $A \cap B$ (ii) $B \cap C$ (iii) $A \cap C \cap D$
(iv) $A \cap C$ (v) $B \cap D$ (vi) $A \cap (B \cup C)$
(vii) $A \cap D$ (viii) $A \cap (B \cup D)$ (ix) $(A \cap B) \cap (B \cup C)$
(x) $(A \cup D) \cap (B \cup C)$

7. If A = {x : x is a natural number }, B = {x : x is an even natural number}
C = {x : x is an odd natural number}andD = {x : x is a prime number }, find
(i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$
(iv) $B \cap C$ (v) $B \cap D$ (vi) $C \cap D$

8. Which of the following pairs of sets are disjoint
(i) {1, 2, 3, 4} and {x : x is a natural number and $4 \le x \le 6$ }
(ii) { a, e, i, o, u } and { c, d, e, f }
(iii) {x : x is an even integer } and {x : x is an odd integer}

9. If A = {3, 6, 9, 12, 15, 18, 21}, B = { 4, 8, 12, 16, 20 },
C = { 2, 4, 6, 8, 10, 12, 14, 16 }, D = {5, 10, 15, 20 }; find
(i) A – B (ii) A – C (iii) A – D (iv) B – A
(v) C – A (vi) D – A (vii) B – C (viii) B – D
(ix) C – B (x) D – B (xi) C – D (xii) D – C

10. If X= { a, b, c, d } and Y = { f, b, d, g}, find
(i) X – Y (ii) Y – X (iii) $X \cap Y$

11. If **R** is the set of real numbers and **Q** is the set of rational numbers, then what is **R** – **Q**?

12. State whether each of the following statement is true or false. Justify your answer.
(i) { 2, 3, 4, 5 } and { 3, 6} are disjoint sets.
(ii) { a, e, i, o, u } and { a, b, c, d }are disjoint sets.
(iii) { 2, 6, 10, 14 } and { 3, 7, 11, 15} are disjoint sets.
(iv) { 2, 6, 10 } and { 3, 7, 11} are disjoint sets.

1.11 Complement of a Set

Let U be the universal set which consists of all prime numbers and A be the subset of U which consists of all those prime numbers that are not divisors of 42. Thus, A = {x : $x \in$ U and x is not a divisor of 42 }. We see that $2 \in$ U but $2 \notin$ A, because 2 is divisor of 42. Similarly, $3 \in$ U but $3 \notin$ A, and $7 \in$ U but $7 \notin$ A. Now 2, 3 and 7 are the only elements of U which do not belong to A. The set of these three prime numbers, i.e., the set {2, 3, 7} is called the *Complement* of A with respect to U, and is denoted by

A′. So we have $A' = \{2, 3, 7\}$. Thus, we see that

$$A' = \{x : x \in U \text{ and } x \notin A\}.$$ This leads to the following definition.

Definition 8 Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A′ to denote the complement of A with respect to U. Thus,

$$A' = \{x : x \in U \text{ and } x \notin A\}. \text{ Obviously } A' = U - A$$

We note that the complement of a set A can be looked upon, alternatively, as the difference between a universal set U and the set A.

Example 20 Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Find A′.

Solution We note that 2, 4, 6, 8, 10 are the only elements of U which do not belong to A. Hence $A' = \{2, 4, 6, 8, 10\}$.

Example 21 Let U be universal set of all the students of Class XI of a coeducational school and A be the set of all girls in Class XI. Find A′.

Solution Since A is the set of all girls, A′ is clearly the set of all boys in the class.

Note If A is a subset of the universal set U, then its complement A′ is also a subset of U.

Again in Example 20 above, we have $A' = \{2, 4, 6, 8, 10\}$

Hence $(A')' = \{x : x \in U \text{ and } x \notin A'\}$

$= \{1, 3, 5, 7, 9\} = A$

It is clear from the definition of the complement that for any subset of the universal set U, we have $(A')' = A$

Now, we want to find the results for $(A \cup B)'$ and $A' \cap B'$ in the followng example.

Example 22 Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.
Find A′, B′, $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.

Solution Clearly $A' = \{1, 4, 5, 6\}$, $B' = \{1, 2, 6\}$. Hence $A' \cap B' = \{1, 6\}$

Also $A \cup B = \{2, 3, 4, 5\}$, so that $(A \cup B)' = \{1, 6\}$

$$(A \cup B)' = \{1, 6\} = A' \cap B'$$

It can be shown that the above result is true in general. If A and B are any two subsets of the universal set U, then

$(A \cup B)' = A' \cap B'$. Similarly, $(A \cap B)' = A' \cup B'$. These two results are stated in words as follows :

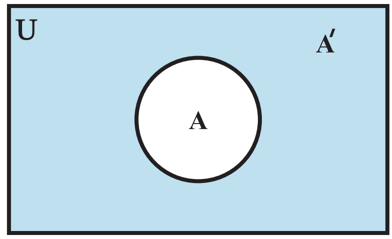


Fig 1.10

The complement of the union of two sets is the intersection of their complements and the complement of the intersection of two sets is the union of their complements. These are called *De Morgan's laws*. These are named after the mathematician De Morgan.

The complement A′ of a set A can be represented by a Venn diagram as shown in Fig 1.10.

The shaded portion represents the complement of the set A.

Some Properties of Complement Sets

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

4. Laws of empty set and universal set $\phi' = U$ and $U' = \phi$.

These laws can be verified by using Venn diagrams.

EXERCISE 1.5

1. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = { 1, 2, 3, 4}, B = { 2, 4, 6, 8 } and C = { 3, 4, 5, 6 }. Find (i) A′ (ii) B′ (iii) $(A \cup C)'$ (iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$

2. If U = { a, b, c, d, e, f, g, h}, find the complements of the following sets :
(i) A = {a, b, c} (ii) B = {d, e, f, g}
(iii) C = {a, c, e, g} (iv) D = { f, g, h, a}

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:
(i) {$x : x$ is an even natural number} (ii) { $x : x$ is an odd natural number }
(iii) {$x : x$ is a positive multiple of 3} (iv) { $x : x$ is a prime number }
(v) {$x : x$ is a natural number divisible by 3 and 5}
(vi) { $x : x$ is a perfect square } (vii) { $x : x$ is a perfect cube}
(viii) { $x : x + 5 = 8$ } (ix) { $x : 2x + 5 = 9$}
(x) { $x : x \geq 7$ } (xi) { $x : x \in$ N and $2x + 1 > 10$ }

4. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = {2, 4, 6, 8} and B = { 2, 3, 5, 7}. Verify that
(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

5. Draw appropriate Venn diagram for each of the following :
(i) $(A \cup B)'$, (ii) $A' \cap B'$, (iii) $(A \cap B)'$, (iv) $A' \cup B'$

6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A′?

7. Fill in the blanks to make each of the following a true statement :

(i) $A \cup A' = \ldots$ (ii) $\phi' \cap A = \ldots$

(iii) $A \cap A' = \ldots$ (iv) $U' \cap A = \ldots$

1.12 Practical Problems on Union and Intersection of Two Sets

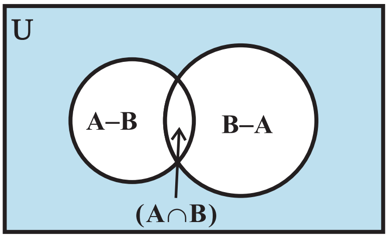


Fig 1.11

In earlier Section, we have learnt union, intersection and difference of two sets. In this Section, we will go through some practical problems related to our daily life.The formulae derived in this Section will also be used in subsequent Chapter on Probability (Chapter 16).

Let A and B be finite sets. If $A \cap B = \phi$, then

(i) $n (A \cup B) = n (A) + n (B)$ ... (1)

The elements in $A \cup B$ are either in A or in B but not in both as $A \cap B = \phi$. So, (1) follows immediately.

In general, if A and B are finite sets, then

(ii) $n (A \cup B) = n (A) + n (B) - n (A \cap B)$ ... (2)

Note that the sets $A - B$, $A \cap B$ and $B - A$ are disjoint and their union is $A \cup B$ (Fig 1.11). Therefore

$$n (A \cup B) = n (A - B) + n (A \cap B) + n (B - A)$$
$$= n (A - B) + n (A \cap B) + n (B - A) + n (A \cap B) - n (A \cap B)$$
$$= n (A) + n (B) - n (A \cap B), \text{ which verifies (2)}$$

(iii) If A, B and C are finite sets, then

$$n (A \cup B \cup C) = n (A) + n (B) + n (C) - n (A \cap B) - n (B \cap C) - n (A \cap C) + n (A \cap B \cap C) \quad \ldots (3)$$

In fact, we have

$$n (A \cup B \cup C) = n (A) + n (B \cup C) - n [A \cap (B \cup C)] \quad [\text{ by (2) }]$$
$$= n (A) + n (B) + n (C) - n (B \cap C) - n [A \cap (B \cup C)] \quad [\text{ by (2) }]$$

Since $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, we get

$$n [A \cap (B \cup C)] = n (A \cap B) + n (A \cap C) - n [(A \cap B) \cap (A \cap C)]$$
$$= n (A \cap B) + n (A \cap C) - n (A \cap B \cap C)$$

Therefore

$$n (A \cup B \cup C) = n (A) + n (B) + n (C) - n (A \cap B) - n (B \cap C) - n (A \cap C) + n (A \cap B \cap C)$$

This proves (3).

Example 23 If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have ?

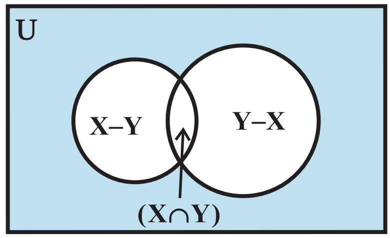


Fig 1.12

Solution Given that

$n(X \cup Y) = 50$, $n(X) = 28$, $n(Y) = 32$,
$n(X \cap Y) = ?$

By using the formula

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y),$$

we find that

$$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$$
$$= 28 + 32 - 50 = 10$$

Alternatively, suppose $n(X \cap Y) = k$, then

$n(X - Y) = 28 - k$, $n(Y - X) = 32 - k$ (by Venn diagram in Fig 1.12)

This gives $50 = n(X \cup Y) = n(X - Y) + n(X \cap Y) + n(Y - X)$
$= (28 - k) + k + (32 - k)$

Hence $k = 10$.

Example 24 In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics ?

Solution Let M denote the set of teachers who teach mathematics and P denote the set of teachers who teach physics. In the statement of the problem, the word 'or' gives us a clue of union and the word 'and' gives us a clue of intersection. We, therefore, have

$$n(M \cup P) = 20, \; n(M) = 12 \text{ and } n(M \cap P) = 4$$

We wish to determine $n(P)$.

Using the result

$$n(M \cup P) = n(M) + n(P) - n(M \cap P),$$

we obtain

$$20 = 12 + n(P) - 4$$

Thus $n(P) = 12$

Hence 12 teachers teach physics.

Example 25 In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football ?

Solution Let X be the set of students who like to play cricket and Y be the set of students who like to play football. Then $X \cup Y$ is the set of students who like to play at least one game, and $X \cap Y$ is the set of students who like to play both games.

Given $n(X) = 24$, $n(Y) = 16$, $n(X \cup Y) = 35$, $n(X \cap Y) = ?$

Using the formula $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we get

$$35 = 24 + 16 - n(X \cap Y)$$

Thus, $n(X \cap Y) = 5$
i.e., 5 students like to play both games.

Example 26 In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Solution Let U denote the set of surveyed students and A denote the set of students taking apple juice and B denote the set of students taking orange juice. Then

$$n(U) = 400,\ n(A) = 100,\ n(B) = 150 \text{ and } n(A \cap B) = 75.$$

Now $n(A' \cap B') = n(A \cup B)'$
$= n(U) - n(A \cup B)$
$= n(U) - n(A) - n(B) + n(A \cap B)$
$= 400 - 100 - 150 + 75 = 225$

Hence 225 students were taking neither apple juice nor orange juice.

Example 27 There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2, and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(i) Chemical C_1 but not chemical C_2 (ii) Chemical C_2 but not chemical C_1

(iii) Chemical C_1 or chemical C_2

Solution Let U denote the universal set consisting of individuals suffering from the skin disorder, A denote the set of individuals exposed to the chemical C_1 and B denote the set of individuals exposed to the chemical C_2.

Here $n(U) = 200,\ n(A) = 120,\ n(B) = 50$ and $n(A \cap B) = 30$

(i) From the Venn diagram given in Fig 1.13, we have

$$A = (A - B) \cup (A \cap B).$$

$n(A) = n(A - B) + n(A \cap B)$ (Since A – B) and $A \cap B$ are disjoint.)

or $n(A - B) = n(A) - n(A \cap B) = 120 - 30 = 90$

Hence, the number of individuals exposed to chemical C_1 but not to chemical C_2 is 90.

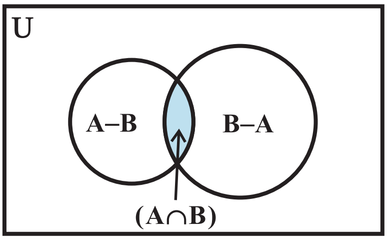


Fig 1.13

(ii) From the Fig 1.13, we have

$$B = (B - A) \cup (A \cap B).$$

and so, $n(B) = n(B - A) + n(A \cap B)$
(Since B – A and $A \cap B$ are disjoint.)
or $n(B - A) = n(B) - n(A \cap B)$
$= 50 - 30 = 20$

Thus, the number of individuals exposed to chemical C_2 and not to chemical C_1 is 20.

(iii) The number of individuals exposed either to chemical C_1 or to chemical C_2, i.e.,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 120 + 50 - 30 = 140.$$

EXERCISE 1.6

1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements ; how many elements does $X \cap Y$ have?

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

5. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

6. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Miscellaneous Examples

Example 28 Show that the set of letters needed to spell " CATARACT " and the set of letters needed to spell " TRACT" are equal.

Solution Let X be the set of letters in "CATARACT". Then

$$X = \{ C, A, T, R \}$$

Let Y be the set of letters in " TRACT". Then

$$Y = \{ T, R, A, C, T \} = \{ T, R, A, C \}$$

Since every element in X is in Y and every element in Y is in X. It follows that X = Y.

Example 29 List all the subsets of the set $\{ -1, 0, 1 \}$.

Solution Let $A = \{ -1, 0, 1 \}$. The subset of A having no element is the empty set ϕ. The subsets of A having one element are $\{ -1 \}$, $\{ 0 \}$, $\{ 1 \}$. The subsets of A having two elements are $\{-1, 0\}$, $\{-1, 1\}$,$\{0, 1\}$. The subset of A having three elements of A is A itself. So, all the subsets of A are ϕ, $\{-1\}$, $\{0\}$, $\{1\}$, $\{-1, 0\}$, $\{-1, 1\}$, $\{0, 1\}$ and $\{-1, 0, 1\}$.

Example 30 Show that $A \cup B = A \cap B$ implies $A = B$

Solution Let $a \in A$. Then $a \in A \cup B$. Since $A \cup B = A \cap B$, $a \in A \cap B$. So $a \in B$. Therefore, $A \subset B$. Similarly, if $b \in B$, then $b \in A \cup B$. Since
$A \cup B = A \cap B$, $b \in A \cap B$. So, $b \in A$. Therefore, $B \subset A$. Thus, $A = B$

Example 31 For any sets A and B, show that
$P(A \cap B) = P(A) \cap P(B)$.

Solution Let $X \in P(A \cap B)$. Then $X \subset A \cap B$. So, $X \subset A$ and $X \subset B$. Therefore, $X \in P(A)$ and $X \in P(B)$ which implies $X \in P(A) \cap P(B)$. This gives $P(A \cap B) \subset P(A) \cap P(B)$. Let $Y \in P(A) \cap P(B)$. Then $Y \in P(A)$ and $Y \in P(B)$. So, $Y \subset A$ and $Y \subset B$. Therefore, $Y \subset A \cap B$, which implies $Y \in P(A \cap B)$. This gives $P(A) \cap P(B) \subset P(A \cap B)$
Hence $P(A \cap B) = P(A) \cap P(B)$.

Example 32 A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both products?

Solution Let U be the set of consumers questioned, S be the set of consumers who liked the product A and T be the set of consumers who like the product B. Given that

$$n(U) = 1000,\ n(S) = 720,\ n(T) = 450$$

So $$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$
$$= 720 + 450 - n(S \cap T) = 1170 - n(S \cap T)$$

Therefore, $n(S \cup T)$ is maximum when $n(S \cap T)$ is least. But $S \cup T \subset U$ implies $n(S \cup T) \leq n(U) = 1000$. So, maximum values of $n(S \cup T)$ is 1000. Thus, the least value of $n(S \cap T)$ is 170. Hence, the least number of consumers who liked both products is 170.

Example 33 Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?

Solution Let U be the set of car owners investigated, M be the set of persons who owned car A and S be the set of persons who owned car B.
Given that $n(U) = 500$, $n(M) = 400$, $n(S) = 200$ and $n(S \cap M) = 50$.
Then $n(S \cup M) = n(S) + n(M) - n(S \cap M) = 200 + 400 - 50 = 550$
But $S \cup M \subset U$ implies $n(S \cup M) \leq n(U)$.
This is a contradiction. So, the given data is incorrect.

Example 34 A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?

Solution Let F, B and C denote the set of men who received medals in football, basketball and cricket, respectively.

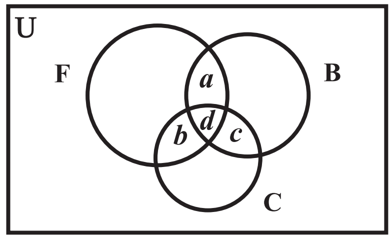


Fig 1.14

Then n (F) = 38, n (B) = 15, n (C) = 20

n (F ∪ B ∪ C) = 58 and n (F ∩ B ∩ C) = 3

Therefore, n (F ∪ B ∪ C) = n (F) + n (B) + n (C) – n (F ∩ B) – n (F ∩ C) – n (B ∩ C) + n (F ∩ B ∩ C),

gives n (F ∩ B) + n (F ∩ C) + n (B ∩ C) = 18

Consider the Venn diagram as given in Fig 1.14

Here, a denotes the number of men who got medals in football and basketball only, b denotes the number of men who got medals in football and cricket only, c denotes the number of men who got medals in basket ball and cricket only and d denotes the number of men who got medal in all the three. Thus, d = n (F ∩ B ∩ C) = 3 and $a + d + b + d + c + d = 18$

Therefore $a + b + c = 9$,

which is the number of people who got medals in exactly two of the three sports.

Miscellaneous Exercise on Chapter 1

1. Decide, among the following sets, which sets are subsets of one and another:
A = { x : x ∈ **R** and x satisfy $x^2 - 8x + 12 = 0$ },
B = { 2, 4, 6 }, C = { 2, 4, 6, 8, . . . }, D = { 6 }.

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If x ∈ A and A ∈ B , then x ∈ B
(ii) If A ⊂ B and B ∈ C , then A ∈ C
(iii) If A ⊂ B and B ⊂ C , then A ⊂ C
(iv) If A ⊄ B and B ⊄ C , then A ⊄ C
(v) If x ∈ A and A ⊄ B , then x ∈ B
(vi) If A ⊂ B and x ∉ B , then x ∉ A

3. Let A, B, and C be the sets such that A ∪ B = A ∪ C and A ∩ B = A ∩ C. Show that B = C.

4. Show that the following four conditions are equivalent :

(i) A ⊂ B(ii) A – B = φ (iii) A ∪ B = B (iv) A ∩ B = A

5. Show that if A ⊂ B, then C – B ⊂ C – A.

6. Assume that P (A) = P (B). Show that A = B

7. Is it true that for any sets A and B, P (A) ∪ P (B) = P (A ∪ B)? Justify your answer.

8. Show that for any sets A and B,
$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = (A \cup B)$

9. Using properties of sets, show that
(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$.

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

11. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that A = B.
(**Hints** $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use Distributive law)

12. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

14. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:
(i) the number of people who read at least one of the newspapers.
(ii) the number of people who read exactly one newspaper.

16. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Summary

This chapter deals with some basic definitions and operations involving sets. These are summarised below:

- A set is a well-defined collection of objects.
- A set which does not contain any element is called *empty set*.
- A set which consists of a definite number of elements is called *finite set*, otherwise, the set is called *infinite set*.
- Two sets A and B are said to be equal if they have exactly the same elements.
- A set A is said to be subset of a set B, if every element of A is also an element of B. Intervals are subsets of **R**.
- A power set of a set A is collection of all subsets of A. It is denoted by P(A).

- The union of two sets A and B is the set of all those elements which are either in A or in B.
- The intersection of two sets A and B is the set of all elements which are common. The difference of two sets A and B in this order is the set of elements which belong to A but not to B.
- The complement of a subset A of universal set U is the set of all elements of U which are not the elements of A.
- For any two sets A and B, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
- If A and B are finite sets such that $A \cap B = \phi$, then
  $n(A \cup B) = n(A) + n(B)$.
  If $A \cap B \neq \phi$, then
  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Historical Note

The modern theory of sets is considered to have been originated largely by the German mathematician Georg Cantor (1845-1918). His papers on set theory appeared sometimes during 1874 to 1897. His study of set theory came when he was studying trigonometric series of the form $a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + ...$ He published in a paper in 1874 that the set of real numbers could not be put into one-to-one correspondence wih the integers. From 1879 onwards, he publishd several papers showing various properties of abstract sets.

Cantor's work was well received by another famous mathematician Richard Dedekind (1831-1916). But Kronecker (1810-1893) castigated him for regarding infinite set the same way as finite sets. Another German mathematician Gottlob Frege, at the turn of the century, presented the set theory as principles of logic. Till then the entire set theory was based on the assumption of the existence of the set of all sets. It was the famous Englih Philosopher Bertand Russell (1872-1970) who showed in 1902 that the assumption of existence of a set of all sets leads to a contradiction. This led to the famous Russell's Paradox. Paul R.Halmos writes about it in his book 'Naïve Set Theory' that "nothing contains everything".

The Russell's Paradox was not the only one which arose in set theory. Many paradoxes were produced later by several mathematicians and logicians.