Arithmetic-Geometric Progression (AGP): This is a sequence in which each term consists of the product of an arithmetic progression and a geometric progression. In variables, It looks like:

$a,(a+d)r,(a+2d)r^2,(a+3d)r^3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,(a+(n-1)d)r^{n-1}$

where a is the first term, d is a common difference, and r is the common ratio.

The general term of AGP: The n_{th} term of the AGP is obtained by multiplying the corresponding terms of the arithmetic progression (AP) and the geometric progression (GP).

So, in the above sequence, the n_{th} term is given by:

$$T_n=(a+(n-1)d)r^{n-1}$$

Sum of terms of AGP : The sum of the first n terms of the AGP is

$$S_n = \sum_{k=1}^{n}(a+(k-1)d)r^{k-1}$$

which can be solved further to obtain

$$S_n = \frac{a-(a\ +(n-1)d)r^n}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2}$$

PROOF:

again using the method of substraction:

$S_n=a+(a+d)r+(a+2d)r^2+(a+3d)r^3+(a+4d)r^4+\ldots\ldots+(a+(n-1)d)r^{n-1}$ eqn_1

$\underline{S_n r= \quad ar+(a+d)r^2+(a+2d)r^3+(a+3d)r^4+(a+4d)r^5+\ldots..+(a+(n-1)d)r^n}$ eqn_2

$S_n-S_n r = a+dr+dr^2+dr^3+dr^4+\cdots+dr^{n-1}-(a+(n-1)d)r^n$

look at the equation : $\boldsymbol{dr+dr^2+dr^3+dr^4+\cdots+dr^{n-1}}$

this is GP with first term dr and and common ratio r

therefore $\boldsymbol{dr+dr^2+dr^3+dr^4+\cdots+dr^{n-1}=\frac{dr(1-r^{n-1})}{1-r}}$

$S_n-S_n r = a+\frac{\mathbf{dr(1-r^{n-1})}}{\mathbf{1-r}}-(a+(n-1)d)r^n$

$$S_n = \frac{\mathbf{dr(1-r^{n-1})}}{\mathbf{(1-r)^2}} + \frac{\mathbf{a-(a+(n-1)d)r^n}}{\mathbf{(1-r)}}$$

Now what if |r|<1 then there is possibility of infinte AGP:

$$\boldsymbol{S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{dr\left(1 - r^{n-1}\right)}{(1-r)^2} + \frac{a - (a + (n-1)d)r^n}{(1-r)}}$$

$$\boldsymbol{= \frac{dr}{(1-r)^2} + \frac{a}{1-r}}$$

[$\boldsymbol{What\ is\ this\ condition\ |r| < 1\ , why\ we\ need\ this\ for\ infinite\ GP\ or\ AGP}$?

$now\ consider\ x = 0.1 = \frac{1}{10}\quad |\mathrm{x}| < 1$

$let's\ calculate\ x^2 = 0.01 = \frac{1}{100}\ < x$

$again\ x^3 = 0.001 = \frac{1}{1000}\ again\ which\ is\ very\ small\ as\ compared\ to\ x$

$now\ observe\ the\ parttern\ , as\ we\ increase\ the\ power\ of\ x\ it'svalue\ was\ getting\ small\ and\ small, what\ if$
$we\ go\ on\ upto\ infinity\ it's\ value\ will\ be\ tending\ (approaches\ )to\ \mathbf{0}$

$So, \lim_{n\to\infty} r^n = 0\ if\ and\ only\ if\ |r| < 1\ \boldsymbol{and}\ \lim_{n\to\infty} r^n \to \infty\ if\ and\ only\ if\ |r| > 1$

$so, |r| < 1\ is\ a\ necessary\ condition\ for\ existance\ of\ infinite\ GP\ or\ AGP.$]

Example: To find the sum $1+2x+3x^2+4x^3+5x^4+$.............. upto infinite terms ($|x|<1$)

Ex Find the sum $1+2x+3x^2+4x^3+\dots\infty$ $(|x|<1)$

Solution: method)1).

let $S = 1+2x+3x^2+4x^3+\dots$

$Sx = x+2x^2+3x^3+\dots$

$S - Sx = 1+x+x^2+x^3+\dots$

$S(1-x) = \frac{1}{1-x}$

$\Rightarrow S = \frac{1}{(1-x)^2}$

method 2)

direct putting all values in the formula

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$$= \frac{1}{1-x} + \frac{x}{(1-x)^2} = \frac{(1-x)+x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

method 3)

let, $S = 1+x+x^2+x^3+\dots\infty$

$1+x+x^2+x^3+\dots\infty = \frac{1}{1-x}$

differentiate both hand side w.r.t. x

$0+1+2x+3x^2+\dots\infty = \frac{1}{(1-x)^2}$

$\therefore 1+2x+3x^2+\dots = \frac{1}{(1-x)^2}$