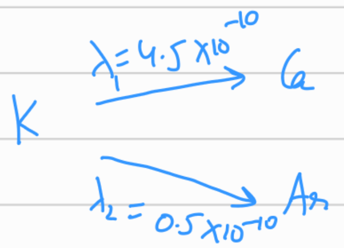
Q: (JEEA 2019) In a radioactive sample, ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constants 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in the sample all the stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t \times 10^{9}$ years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be,
[Given $\ln 10 = 2.3$]

a) 1.15 b) 9.2

c) 2.3 d) 4.6



$\lambda_{eff} = \lambda_1 + \lambda_2$

Let initial no. of K nuclei be N_0.

then, $N_K = N_0 e^{-\lambda_{eff} t}$

Rate of production of Ca $= \lambda_1 N_K$

On integrating

$$N_{Ca} = \frac{\lambda_1 N_0 (1 - e^{-\lambda_{eff} t})}{\lambda_{eff}}$$

Similarly $$N_{Ar} = \frac{\lambda_2 N_0 (1 - e^{-\lambda_{eff} t})}{\lambda_{eff}}$$

Given $$\frac{N_{Ca} + N_{Ar}}{N_K} = 99$$

$$\Rightarrow \frac{(\lambda_1 + \lambda_2) N_0 (1 - e^{-\lambda_{eff} t})}{\lambda_{eff} \times N_0 e^{-\lambda_{eff} t}} = 99$$

$$\Rightarrow 1 = 100 e^{-\lambda_{eff} t}$$

$$100 = e^{\lambda_{eff} t} \Rightarrow 2 \ln 10 = 5 \times 10^{-10} \times t \times 10^{9}$$

$$t = 4 \ln 10 = \underline{9.2}$$

Alternate method,

Sum of nuclei of Ca and Ar = No. of K nuclei disintegrated

$= N_0 - N$

$$\frac{N_0 - N}{N} = 99$$

$N_0 = 100N$

$N_0 = 100 N_0 e^{-\lambda_{eff} t}$

$\lambda_{eff} \times t = \ln 100$

$\Rightarrow 5 \times 10^{-10} \times t \times 10^9 = 2 \times 2.3$

$\Rightarrow t = 9.2$