. *A small particle of mass m moves in such a way that the potential energy* $U = \frac{1}{2} m^2 \omega^2 r^2$ *where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the nth allowed orbit is proportional to* $\sqrt{n}$.

Solution : The force at a distance r is

$$F = -\frac{dU}{dr} = -m\omega^2 r. \qquad \dots \text{(i)}$$

Suppose the particle moves along a circle of radius r. The net force on it should be mv^2/r along the radius. Comparing with (i),

$$\frac{mv^2}{r} = m\omega^2 r$$

or, $$v = \omega r. \qquad \dots \text{(ii)}$$

The quantization of angular momentum gives

$$mvr = \frac{nh}{2\pi}$$

or, $$v = \frac{nh}{2\pi mr}. \qquad \dots \text{(iii)}$$

From (ii) and (iii),

$$r = \left(\frac{nh}{2\pi m\omega}\right)^{1/2}.$$

Thus, the radius of the nth orbit is proportional to $\sqrt{n}$.