

A neutron moving with speed  $v$  makes a head-on collision with a hydrogen atom in ground state kept at rest. Find the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron  $\approx$  mass of hydrogen  $= 1.67 \times 10^{-27}$  kg.

**Solution :** Suppose the neutron and the hydrogen atom move at speeds  $v_1$  and  $v_2$  after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy  $\Delta E$  is used in this way. Using conservation of linear momentum and energy,

$$mv = mv_1 + mv_2 \quad \dots \text{ (i)}$$

and 
$$\frac{1}{2} mv^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 + \Delta E \quad \dots \text{ (ii)}$$

From (i), 
$$v^2 = v_1^2 + v_2^2 + 2v_1v_2.$$

From (ii), 
$$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}.$$

Thus, 
$$2v_1v_2 = \frac{2\Delta E}{m}.$$

Hence, 
$$(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}.$$

As  $v_1 - v_2$  must be real,

$$v^2 - \frac{4\Delta E}{m} \geq 0$$

or, 
$$\frac{1}{2} mv^2 > 2\Delta E.$$

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state

is  $10.2 \text{ eV}$ . Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is

$$\frac{1}{2} m v_{\min}^2 = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV}.$$