. *Derive an expression for the magnetic field at the site of the nucleus in a hydrogen atom due to the circular motion of the electron. Assume that the atom is in its ground state and give the answer in terms of fundamental constants.*

Solution : We have

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

or, $$v^2 r = \frac{e^2}{4\pi\varepsilon_0 m} \qquad \dots \text{(i)}$$

From Bohr's quantization rule, in ground state,

$$vr = \frac{h}{2\pi m} \qquad \dots \text{(ii)}$$

From (i) and (ii),

$$v = \frac{e^2}{2\varepsilon_0 h} \qquad \dots \text{(iii)}$$

and $$r = \frac{\varepsilon_0 h^2}{\pi m e^2} \qquad \dots \text{(iv)}$$

As the electron moves along a circle, it crosses any point on the circle $\frac{v}{2\pi r}$ times per unit time. The charge crossing per unit time, that is the current, is $i = \frac{ev}{2\pi r}$. The magnetic field at the centre due to this circular current is

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 ev}{4\pi r^2}.$$

From (iii) and (iv),

$$B = \frac{\mu_0 e}{4\pi} \frac{e^2}{2\varepsilon_0 h} \times \frac{\pi^2 m^2 e^4}{\varepsilon_0^2 h^4}$$

$$= \frac{\mu_0 e^7 \pi m^2}{8\varepsilon_0^3 h^5}.$$