Q3. Sample of two radioactive nuclides A and B are taken. λ_A and λ_B are the disintegration constants of A and B respectively. In which of the following cases, the two samples can simultaneously have the same decay rate at any time?

a) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$.

b) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$.

c) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$.

d) Initial rate of decay of B is same as the rate of decay of A at $t = 2h$ and $\lambda_B < \lambda_A$.

A3.

Rate of decay, $R = \lambda N_0 e^{-\lambda t}$ where N_0 = initial no. of nuclei

λ = decay constant

intial rate of decay, $R(t=0) = \lambda N_0 = R_0$

$R_A = R_B$

$R_{0A} e^{-\lambda_A t} = R_{0B} e^{-\lambda_B t}$ for some t_0. We have to find such t_0.

(a) $R_{0A} = 2R_{0B}$

$\lambda_A = \lambda_B = \lambda$ (say)

$$R_{0A} e^{-\lambda_A t_0} = R_{0B} e^{-\lambda_B t_0}$$

$$2R_{0B} e^{-\lambda t_0} = R_{0B} e^{-\lambda t_0}$$

No such t_0 is possible ✗

(b) $R_{0A} = 2R_{0B}$

$$R_{0A} e^{-\lambda_A t_0} = R_{0B} e^{-\lambda_B t_0}$$

$$2R_{0B} e^{-\lambda_A t_0} = R_{0B} e^{-\lambda_B t_0}$$

$$2 = e^{(\lambda_A - \lambda_B)t_0}$$

$$t_0 = \frac{\ln 2}{\lambda_A - \lambda_B}$$ since $\lambda_A > \lambda_B$, t_0 is positive. Hence, a solution exists

✓

(c) $R_{0B} = 2R_{0A}$

$$R_{0A} e^{-\lambda_A t_0} = R_{0B} e^{-\lambda_B t_0}$$

$$\cancel{R_{0A}} e^{-\lambda_A t_0} = 2\cancel{R_{0A}} e^{-\lambda_B t_0}$$

$$e^{(\lambda_B - \lambda_A)t_0} = 2$$

$$t_0 = \frac{\ln 2}{\lambda_B - \lambda_A}$$ $\lambda_A > \lambda_B \Rightarrow t_0$ is negative

$\Rightarrow$ No valid solution.

X

(d) $R_{0B} = R_A(t=2)$

$$R_A = R_{0A} e^{-2\lambda_A} = R_{0B}$$

$$R_{0A} e^{-\lambda_A t_0} = R_{0B} e^{-\lambda_B t_0}$$

$$\cancel{R_{0A}} e^{-\lambda_A t_0} = \cancel{R_{0A}} e^{-2\lambda_A} e^{-\lambda_B t_0}$$

$$e^{(2-t_0)\lambda_A} = e^{-\lambda_B t_0}$$

$$(t_0 - 2)\lambda_A = \lambda_B t_0$$

$$t_0 \lambda_A - 2\lambda_A = \lambda_B t_0$$

$$t_0 = \frac{2\lambda_A}{\lambda_A - \lambda_B}$$, $\lambda_A > \lambda_B \Rightarrow t_0 > 0$

$\Rightarrow$ Solution is valid

✓

Answer = b, d