

Notes

Taylor Series \Rightarrow

Taylor series is the polynomial or a function of an infinite sum of items. Each successive term will have a larger exponent or higher degree than the preceding term.

$$f(x) = f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \dots$$

$f'(a)$, $f''(a)$, $f'''(a)$ are derivatives at point a of $f(x)$. If the value of a is 0 (zero), then the Taylor series is also called **Maclaurin Series**.

In the sigma notation -

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x-a)^n$$

where $f^n(a) = n^{\text{th}}$ derivative of f
 $n! = \text{factorial of } n$

Taylor Series for $\sin(x)$ \Rightarrow

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

Taylor Series in Several Variables \Rightarrow

$$T(x_1, x_2, x_3, \dots, x_m) = f(a_1, a_2, \dots, a_m) + \sum_{j=1}^m \frac{\partial f(a_1, a_2, \dots, a_m)}{\partial x_j} (x_j - a_j) + \frac{1}{2!} \sum_{j=1}^m \dots$$

Euler's Constant $\Rightarrow (e)$

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$e \approx 2.7$$

$\left(1 + \frac{1}{n}\right)^n$ k^{th} term

for $n > k$

$$\frac{n(n-1)\cdots(n-k+1)}{k!} \left(\frac{1}{n}\right)^k$$

$$= \binom{n}{k} \left(\frac{1}{n}\right)^k$$