NOTES – ECHELON FORM

Row Echelon form of a matrix

a matrix is in **row echelon form** if

- All rows consisting of only zeroes are at the bottom.
- The leading coefficient (also called the pivot) of a nonzero row is always strictly to the right of the leading coefficient of the row above it.

and transpose of row echelon form matrix is column echelon form matrix.

Reduced Row Echelon form (RRE)

A matrix is in **reduced row echelon form** (also called **row canonical form**) if it satisfies the following conditions:[3]

- It is in row echelon form.
- The leading entry in each nonzero row is a 1 (called a leading 1).
- Each column containing a leading 1 has zeros in all its other entries.

Transformation to Row Echelon form

By means of a finite sequence of elementary row operations, called Gaussian elimination, any matrix can be transformed to row echelon form. Since elementary row operations preserve the row space of the matrix, the row space of the row echelon form is the same as that of the original matrix.

The resulting echelon form is not unique; any matrix that is in echelon form can be put in an (equivalent) echelon form by adding a scalar multiple of a row to one of the above rows.

However, every matrix has a unique *reduced* row echelon form.

This means that the nonzero rows of the reduced row echelon form are the unique reduced row echelon generating set for the row space of the original matrix.

Operation for inversibility

1. $R_i \longleftrightarrow R_j$ $\quad (C_i \longleftrightarrow C_j)$

2. $R_i \longrightarrow KR_i$ $\quad (C_i \longrightarrow \lambda C_j)$

3. $R_i \longrightarrow R_i + KR_j$ $\quad (C_i \longrightarrow C_i + \lambda C_j)$

Equivalent Matrix:-
If a matrix B from a matrix A by one or more elementry transformation then A, B are called equivalent Matrix
We denote by $A \sim B$ [A is equivalent to B]

Eg:- $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$. Find A^{-1}.

$\because \quad A = IA$

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -3 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Here we apply operation to convert A to I and I to some matrix C and since ~~A~~ after ~~after~~ applying operation we get $I = CA$. Therefore C is A^{-1}.

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -3 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad \left(\text{Apply } R_2 \to R_2 - \frac{2R_1}{3}\right)$$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & \frac{2}{3} & -\frac{5}{3} \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad \left(\text{Apply } R_3 \to R_3 - R_1\right)$$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & \frac{2}{3} & -\frac{5}{3} \\ 0 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \quad \left(\text{Apply } R_1 \to R_1 + R_3\right)$$

$$\begin{bmatrix} 3 & -5 & 0 \\ 0 & \frac{2}{3} & -\frac{5}{3} \\ 0 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{2}{3} & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \quad \left(\text{Apply } R_2 \to R_2 + \frac{5}{6} R_3\right)$$

$$\begin{bmatrix} 3 & -5 & 0 \\ 0 & -\frac{8}{3} & 0 \\ 0 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{3}{2} & 1 & \frac{5}{6} \\ -1 & 0 & 1 \end{bmatrix} A \quad \left(\text{Apply } R_2 \to 3R_2\right)$$

$$\begin{bmatrix} 3 & -5 & 0 \\ 0 & -8 & 0 \\ 0 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{9}{2} & 3 & \frac{5}{2} \\ -1 & 0 & 1 \end{bmatrix} A \quad \left(\text{Apply } R_3 \to R_3 - \frac{R_2}{2}\right)$$

$$\begin{bmatrix} 3 & -5 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{9}{2} & 3 & \frac{5}{2} \\ \frac{5}{4} & -\frac{3}{2} & -\frac{1}{4} \end{bmatrix} A \quad \left(\text{Apply } R_2 \to -\frac{R_2}{8},\ R_3 \to \frac{R_3}{2}\right)$$

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$$\begin{bmatrix} -3 & -5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{9}{16} & -\frac{3}{8} & -\frac{5}{16} \\ \frac{5}{8} & -\frac{3}{4} & -\frac{1}{8} \end{bmatrix} A \quad \left(\text{Apply } R_1 \to R_1 + 5R_2\right)$$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{45}{16} & -\frac{15}{8} & -\frac{25}{16} \\ \frac{9}{16} & -\frac{3}{8} & -\frac{5}{16} \\ \frac{5}{8} & -\frac{3}{4} & -\frac{1}{8} \end{bmatrix} A \quad \left(\text{Apply } R_1 \to -\frac{R_1}{3}\right)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{25}{16} & +\frac{5}{8} & +\frac{25}{48} \\ \frac{9}{16} & -\frac{3}{8} & -\frac{5}{16} \\ \frac{5}{8} & -\frac{3}{4} & -\frac{1}{8} \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} -\frac{25}{16} & \frac{5}{8} & \frac{25}{48} \\ \frac{9}{16} & -\frac{3}{8} & -\frac{5}{16} \\ \frac{5}{8} & -\frac{3}{4} & -\frac{1}{8} \end{bmatrix}$$