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① Let A, B are symmetric matrix (non-singular) such that $AB = BA$ then show that
(i) $A^{-1}B^{-1}$ (ii) $A^{-1}B$ are symmetric Matrix

② Let A, B, C are 3×3 mat such that $|\text{adj } A| = 4$, $|B| = 15$, $|C| = 5$ find $|A^2 B C^{-1}|$ (12)

③ Let $|A|_{3\times3} = 2$ & $|B|_{3\times3} = 3$. Find value of $\dfrac{|\text{adj } AB|}{|A^2|\,|\text{adj } B|}$ ①

④ Let $|A|_{3\times3} = 5$ find value of $|3 \text{ adj } 2A|$ 43200

⑤ Let A, B, C, D are non singular Mat same order then show that $(A^{-1}BC^{-1}D)^{-1}(BD^{-1}CB^{-1})^2 \cdot (BC^{-1})^3((B^{-1}C)^{-1})^2 = A^{-1}D^{-1}$

⑥ Let $I+A$ is a non singular Mat. & A is a skew sym. matric then show that $(I-A)(I+A)^{-1}$ is orthogonal Mat.

⑦ Let $|A|_{n\times n} = 3$. Find value of $\left|\text{adj}\left(|A|\,\text{adj } A^{-1}\right)\right|$ $\boxed{3^{n-1}}$

⑧ Let A, B are non-singular square mat of same order then show that $|I - AB| = |I - BA|$

⑨ Let A be a square matrix of order 5 & $9A^{-1} = 4A^T$. Find No. of digits in $|\text{adj adj adj } A|$ (57) $\left(\frac{3}{2}\right)^{320}$

⑩ Let $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 2k-1 & k \\ 1-2k & 0 & 2\sqrt{k} \\ -k & -2\sqrt{k} & 0 \end{bmatrix}$

If $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$, Find k. $\boxed{\frac{9}{2}}$

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(11) Let $A = \begin{bmatrix} 0 & 0 & a \\ 0 & b & c \\ d & e & f \end{bmatrix}$ where $abd \neq 0$ Find No. of matri[x]

$\boxed{8}$

so that $A^{-1} = A^T$

Ans 1 (i) $\frac{\text{adjoint}(AB)}{|AB|} = \frac{\text{adjoint}(BA)}{|B||A|} = A^{-1} \cdot B^{-1} = (BA)^{-1}$

$(AB)^{-1} = (BA)^{-1}$

$(B^{-1}A^{-1}) = ((BA)^{-1}) \Rightarrow AB = B^{-1}A^{-1} = (AB)$

$A^{-1}B^{-1}$ = To prove $A^{-1}B^{-1} = B^{-1}A^{-1}$

Hence proved

(ii) We know that $(A^{-1})^T = (A^T)^{-1} = A^{-1}$

To prove $A^{-1}B = BA^{-1}$

$(AB)^{-1} = (BA)^{-1}$ $\quad A^{-1}B^{-1}B = B^{-1}A^{-1}B$

$BA^{-1} = BB^{-1}A^{-1}B$

$\therefore BA^{-1} = A^{-1}B$

Hence proved

Ans 2 $|A|^{n-1} = 4 \therefore |A|^2 = 4$

$|A^2BC^{-1}| = |A|^2|B||C^{-1}| = 4 \times 15 \times \frac{1}{5} = \boxed{12}$

Ans 3 $\frac{|\text{adjoint}(AB)|}{|A|^2|\text{adjoint}B|} = \frac{|\text{adjoint}B| \cdot |\text{adjoint}A|}{|\text{adjoint}B| \cdot |A|^2} = \frac{|A|^{n-1}}{|A|^2} = \frac{|A|^3}{|A|^2}$

$= \boxed{1}$

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Ans 4 $3\text{ adjoint } 2A = 3 \cdot 2^{3-1} = 3 \cdot 4 \text{ adjoint } A = 12 \text{ adjoint } A$

~~12 adjo~~ $|12 \text{ adjoint } A| = (12)^3 \cdot |A|^2 = (12)^3 \times 25 = \boxed{43200}$

Ans 5 $(A^{-1}BC^{-1}D)(D^{-1}CB^{-1})^2(BC^{-1})^3((B^{2}C^{-1})^{-1})^2$

$\Rightarrow A^{-1}\cancel{BC^{-1}D}\cancel{D^{-1}}\cancel{CB^{-1}}D^{-1}CB^{-1}BC^{-1}\cancel{BC^{-1}BC^{-1}C}BC B^{-1}$

$A^{-1}D^{-1}CB^{-1}BC^{-1}$

$= A^{-1}D^{-1}$

Hence proved

Ans 6 we need to prove $A^{-1} = A^T$

$\therefore ((I-A)(I+A)^{-1})^{-1} = (I+A)(I-A)^{-1}$

$((I-A)(I+A)^{-1})^T = (I-A)^{-1}(I+A)$

need to prove $(I+A)(I-A)^{-1} = (I-A)^{-1}(I+A)$

$(I-A)(I+A)(I-A)^{-1}(I-A) = (I-A)(I-A)^{-1}(I+A)(I-A)$

$\therefore (I-A)(I+A) = (I+A)(I-A)$

$\therefore I^2 - A^2 = I^2 - A^2$

Hence proved

Ans 7 $\left| \text{adj}\left(\dfrac{\text{adjoint}(A^{-1})}{\frac{1}{|A|}} \right) \right|$ $\qquad |A^{-1}| = \dfrac{1}{|A|}$

$\therefore \left| \text{adj}(\text{adjoint}(A^{-1})^{-1}) \right| = |\text{adjoint}(A)|$

$= A^{n-1} = \boxed{3^{n-1}}$

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Ans 8 $CA = ~~B~~ - ABA$

$AD = AI - ~~B~~A$

$\therefore CA = AD$

$\therefore |CA| = |AD|$

$\therefore \boxed{|C| = |D|}$

Hence proved

Ans 9 $|9A^{-1}| = |4A|$

$= \frac{(9)^5}{(4)^5} = |A|^2$

& $|adj(adj A(adj^2 A))| = |A|^{64}$

$= \left(\frac{9}{4}\right)^{5 \times 32} = \boxed{\left(\frac{3}{2}\right)^{320}}$

Ans 10 $|A|^2 + |B|^2 = 10^6$

$|A| = (2k-1)(4k^2-1) - 2\sqrt{k}(-2\sqrt{k} - 4k^{\frac{3}{2}}) + 2\sqrt{k}(4k^{\frac{3}{2}} + 2\sqrt{k})$

$|B| = -(2k-1)2k^{3/2} + 2k^{3/2}(2k-1) = 0$

$|A| = (2k-1)^2(2k+1) + (4k + 8k^2)$

$= (2k-1)^2(2k+1) + 8(2k+1)$

$(4k^2+4k+1)(2k+1) = |A|$

$(2k+1)^3 = 10^6$

$2k+1 = 10$

$\boxed{k = \frac{9}{2}}$

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Ans 11) $A^{-1} = A^T$

$\therefore A \cdot A^T = I$

$$\begin{bmatrix} 0 & 0 & a \\ 0 & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2 & ac & af \\ ac & b^2+c^2 & be+cf \\ af & be+cf & d^2+e^2+f^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c = f = 0 = e$

$\therefore ac = af = be + cf = 0$

& $a^2 = 1$, $b^2 + c^2 = 1$, $d^2 + e^2 + f^2 = 1$

$a \neq 0$, $b \neq 0$, $d \neq 0$

$a = \pm 1$, $b = \pm 1$, $d = \pm 1$

total cases $(2)^3 = \boxed{8}$