The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$ is

(1) 210

(2) 330

(3) 120

(4) 420

Ans. (2)

Solution. We have

$(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$

$$= (1+x)^{10} \frac{\left[1 - \left(\dfrac{x}{1+x}\right)^{11}\right]}{1 - \left(\dfrac{x}{1+x}\right)} = \frac{(1+x)^{10}[(1+x)^{11} - x^{11}]}{(1+x)^{11} \times \dfrac{1}{(1+x)}} = (1+x)^{11} - x^{11}$$

Therefore, the coefficient of x^7 in the expression is

$$^{11}C_7 = {}^{11}C_4 = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = 330$$