

The value of $({}^{21}C_1 \cdot {}^{10}C_1) + ({}^{21}C_2 \cdot {}^{10}C_2) + ({}^{21}C_3 \cdot {}^{10}C_3) + ({}^{21}C_4 \cdot {}^{10}C_4) + \dots + ({}^{21}C_{10} \cdot {}^{10}C_{10})$ is:

- (1) $2^{20} - 2^{10}$
- (2) $2^{21} - 2^{11}$
- (3) $2^{21} - 2^{10}$
- (4) $2^{20} - 2^9$

Ans. (1)

Solution.

$$\begin{aligned} {}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{10} &= 1/2 \left\{ {}^{21}C_0 + {}^{21}C_1 + \dots + {}^{21}C_{21} \right\} - 1 \\ &= 2^{20} - 1 \\ ({}^{10}C_1 + {}^{10}C_2 + \dots + {}^{10}C_{10}) &= 2^{10} - 1 \\ \therefore \text{Required sum} &= (2^{20} - 1) - (2^{10} - 1) \\ &= 2^{20} - 2^{10} \end{aligned}$$