

If ${}^{20}C_1 + (2^2) {}^{20}C_2 + (3^2) {}^{20}C_3 + \dots + (20^2) {}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, P) is equal to:

(1) (420, 19)

(2) (420, 18)

(3) (380, 18)

(4) (380, 19)

Ans. (2)

Solution.

$$\text{Given, } {}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \dots + 20^2 \cdot {}^{20}C_{20} = A(2^\beta)$$

Taking L.H.S.,

$$= \sum_{r=1}^{20} r^2 \cdot {}^{20}C_r = 20 \sum_{r=1}^{20} r \cdot {}^{19}C_{r-1}$$

$$= 20 \left[\sum_{r=1}^{20} (r-1) {}^{19}C_{r-1} + \sum_{r=1}^{20} {}^{19}C_{r-1} \right]$$

$$= 20 \left[19 \sum_{r=2}^{20} {}^{18}C_{r-2} + 2^{19} \right] = 20[19 \cdot 2^{18} + 2^{19}]$$

$$= 420 \times 2^{18}$$

Now, compare it with R.H.S., $A = 420$ and $\beta = 18$