

The sum of the series $2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + 11 \cdot {}^{20}C_3 + \dots + 62 \cdot {}^{20}C_{20}$ is equal to:

(1) 2^{26}

(2) 2^{25}

(3) 2^{23}

(4) 2^{24}

Ans. (2)

Solution.

$$\begin{aligned} & 2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + \dots + 62 \cdot {}^{20}C_{20} \\ &= \sum_{r=0}^{20} (3r+2) {}^{20}C_r = 3 \sum_{r=0}^{20} r \cdot {}^{20}C_r + 2 \sum_{r=0}^{20} {}^{20}C_r \\ &= 60 \sum_{r=1}^{20} {}^{19}C_{r-1} + 2 \sum_{r=0}^{20} {}^{20}C_r \\ &= 60 \times 2^{19} + 2 \times 2^{20} = 2^{21} [15 + 1] = 2^{25} \end{aligned}$$