

Find the coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$

Sol. Given expression is $\left(x^4 - \frac{1}{x^3}\right)^{15}$

$$\therefore T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(-\frac{1}{x^3}\right)^r = {}^{15}C_r x^{60-4r} (-1)^r x^{-3r} = {}^{15}C_r x^{60-7r} (-1)^r$$

For the coefficient x^{-17} , we have

$$60 - 7r = -17 \Rightarrow 7r = 77 \Rightarrow r = 11$$

$$\therefore T_{11+1} = {}^{15}C_{11} x^{60-77} (-1)^{11}$$

$$\begin{aligned}\therefore \text{Coefficient of } x^{-17} &= \frac{-15 \times 14 \times 13 \times 12 \times 11!}{11! \times 4 \times 3 \times 2 \times 1} \\ &= -15 \times 7 \times 13 = -1365\end{aligned}$$