Question 1: Find the value of

$$\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}+\ldots\ldots+\binom{30}{20}\binom{30}{30}$$

Solution:

$(1 - x)^{30} = {}^{30}C_0x_0 - {}^{30}C_1x_1 + {}^{30}C_2x^2 + \ldots\ldots + (-1)^{30}\,{}^{30}C_{30}x^{30}$ ....(i)

$(x + 1)^{30} = {}^{30}C_0\,x^{30} + {}^{30}C_1x^{29} + {}^{30}C_2x^{28} + \ldots\ldots + {}^{30}C_{10}x^{20} + \ldots. + {}^{30}C_{30}x^0$ ....(ii)

Multiplying (i) and (ii) and equating the coefficient of x^{20} on both sides, we get required sum = coefficient of x^{20} in $(1 - x^2)^{30} = {}^{30}C_{10}$.