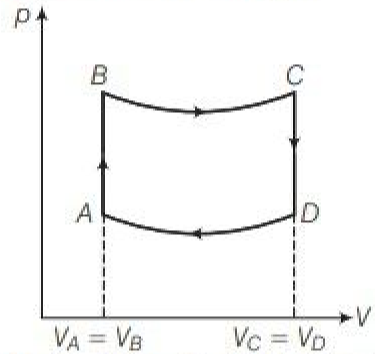
Q. 24 A cycle followed by an engine (made of one mole of perfect gas in a cylinder with a piston) is shown in figure.



A to *B* volume constant, *B* to *C* adiabatic, *C* to *D* volume constant and *D* to *A* adiabatic

$$V_C = V_D = 2V_A = 2V_B$$

(a) In which part of the cycle heat is supplied to the engine from outside?

(b) In which part of the cycle heat is being given to the surrounding by the engine?

(c) What is the work done by the engine in one cycle? Write your answer in term of p_A, p_B, V_A?

(d) What is the efficiency of the engine?

$$\left(\gamma = \frac{5}{3} \text{ for the gas}\right), \left(C_V = \frac{3}{2}R \text{ for one mole}\right)$$

Ans. (a) For the process *AB*,

$$dV = 0 \Rightarrow dW = 0 \quad (\because \text{volume is constant})$$

$$dQ = dU + dW = dU$$

$\Rightarrow$ $dQ = dU$ = Change in internal energy.

Hence, in this process heat supplied is utilised to increase, internal energy of the system.

Since, $p = \left(\frac{nR}{V}\right)T$, in isochoric process, $T \propto p$. So temperature increases with increases of pressure in process *AB* which inturn increases internal energy of the system *i.e.*, $dU > 0$. This imply that $dQ > 0$. So heat is supplied to the system in process *AB*.

(b) For the process *CD*, volume is constant but pressure decreases.
Hence, temperature also decreases so heat is given to surroundings.

(c) To calculate work done by the engine in one cycle, we calculate work done in each part separately.

$$W_{AB} = \int_A^B p dV = 0, \; W_{CD} = \int_{V_C}^{V_D} p dV = 0 \quad (\because dV = 0)$$

$$W_{BC} = \int_{V_B}^{V_C} p dV = k\int_{V_B}^{V_C} \frac{dV}{V^{\gamma}} = \frac{k}{1-\gamma}[V^{1-\gamma}]_{V_B}^{V_C}$$

$$= \frac{1}{1-\gamma}[pV]_{V_B}^{V_C} = \frac{(p_C V_C - p_B V_B)}{1-\gamma}$$

Similarly, $$W_{DA} = \frac{p_A V_A - p_D V_D}{1-\gamma} \quad [\because BC \text{ is adiabatic process}]$$

$\because$ *B* and *C* lies on adiabatic curve *BC*.

$\therefore$ $$p_B V_B^{\gamma} = p_C V_C^{\gamma}$$

$$p_C = p_B \left(\frac{V_B}{V_C}\right)^{\gamma} = p_B\left(\frac{1}{2}\right)^{\gamma} = 2^{-\gamma}\ p_B$$

Similarly, $$p_D = 2^{-\gamma} p_A$$

Total work done by the engine in one cycle *ABCDA*.

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA} = W_{BC} + W_{DA}$$

$$= \frac{(p_C V_C - p_B V_B)}{1-\gamma} + \frac{(p_A V_A - p_D V_D)}{1-\gamma}$$

$$W = \frac{1}{1-\gamma}[2^{-\gamma} p_B (2V_B) - p_B V_B + p_A V_A - 2^{-\gamma} p_B (2V_B)]$$

$$= \frac{1}{1-\gamma}[p_B V_B (2^{-\gamma+1} - 1) - p_A V_A (2^{-\gamma+1} - 1)$$

$$= \frac{1}{1-\gamma}(2^{1-\gamma} - 1)(p_B - p_A)V_A$$

$$= \frac{3}{2}\left[1 - \left(\frac{1}{2}\right)^{2/3}\right](p_B - p_A)V_A$$