Important formulas

$\Rightarrow$ If $a \neq b$ & $a, b > 0$, then

$$\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

$$\boxed{AM \geq G.M \geq H.M} \; ***$$

{ AM = arithmetic mean ; GM = Geometric " ; HM = Harmonic " }

$$\Rightarrow \frac{m_1 a_1 + m_2 a_2 + m_3 a_3 + \cdots + m_n a_n}{m_1 + m_2 + m_3 + \cdots + m_n} \geq \left(a_1^{m_1} \cdot a_2^{m_2} \cdot a_3^{m_3} \cdots a_n^{m_n}\right)^{\frac{1}{m_1+m_2+\cdots m_n}}$$

$$\Rightarrow (1+a_1)(1+a_2)\cdots(1+a_n) > 1+a_1+a_2+\cdots+a_n$$

$$\Rightarrow (a_1 b_1 + a_2 b_2 + \cdots + a_n b_n)^2 \leq (a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)$$

iff $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n}$

$$\Rightarrow n(a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n) \geq (a_1 + a_2 + \cdots + a_n)(b_1 + b_2 + \cdots + b_n)$$

if $a_1 \leq a_2 \leq \cdots \leq a_n$ & $b_1 \leq b_2 \leq \cdots \leq b_n$.

$$\Rightarrow (1-a_1)(1-a_2)(1-a_3)\cdots(1-a_n) > 1-a_1-a_2-a_3 \cdots -a_n$$