

Exemplar problems

Q) The number of real roots of $x^8 - x^5 + x^2 - x + 1 = 0$ is equal to ?

- ⓐ 0 ⓑ 2 ⓒ 4 ⓓ 6

sol. Given equation:

$$\text{let } f(x) = x^8 - x^5 + x^2 - x + 1$$

case (i) for $x < 0$

$f(x) > 0$ [since odd power terms have -ve sign]

so all terms are +ve.

case (ii) for $0 < x < 1$

$$f(x) = \underbrace{x^8}_{+ve} - \underbrace{(x^5 - x^2)}_{+ve} - \underbrace{(x-1)}_{+ve}$$

$\therefore f(x) > 0 \quad \forall 0 < x < 1$

case (iii) for $x = 1 \Rightarrow f(1) = 1 > 0$

case (iv) for $x > 1$

$$f(x) = \underbrace{(x^8 - x^5)}_{+ve} + \underbrace{(x^2 - x)}_{+ve} + 1$$

$\therefore f(x) > 0 \quad \forall x > 1$

Therefore $\forall x \in \mathbb{R}, f(x) > 0$

$\rightarrow f(x) = 0$ has no real roots since $f(x)$ doesn't pass through x -axis.