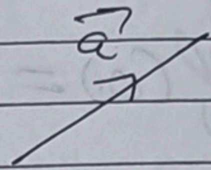
Vectors

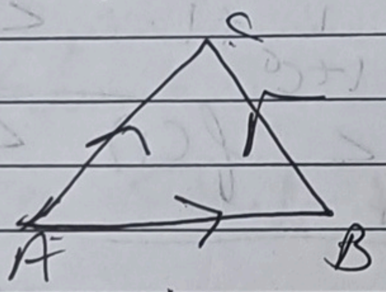
☆ $\hat{a} = \dfrac{\vec{a}}{|a|}$



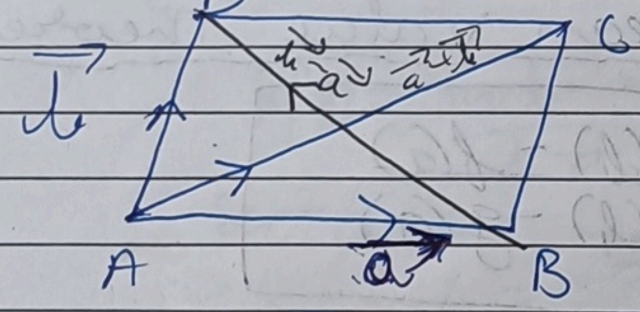
☆ Zero vector : Magnitude zero and direction not defined.
$\vec{AA}$, $\vec{BB}$

☆ Triangle Law of vector addition

$\vec{AB} + \vec{BC} = \vec{AC}$



☆ Parallelogram Law of vector Addition



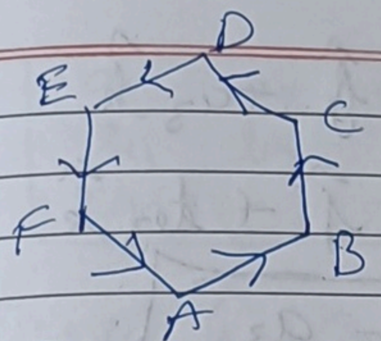
$\vec{AC} = \vec{a} + \vec{b}$

$\vec{AB} + \vec{BD} = \vec{AD}$

$\vec{BD} = \vec{AD} - \vec{AB}$

$\boxed{\vec{BD} = \vec{b} - \vec{a}}$

☆ Polygon Law of vector addition



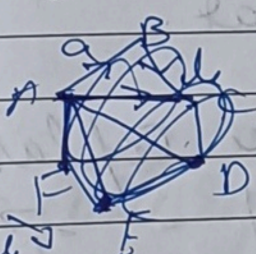
$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} + \vec{FA} = \vec{0}$

Q- In Regular Hexagon ABCDEF

$\vec{AB} = \vec{a}$ $\vec{BC} = \vec{b}$ $CD = \vec{c}$

Find $\vec{AD}$, $\vec{AE}$, $\vec{AF}$

$\vec{a} + \vec{b} = AC$

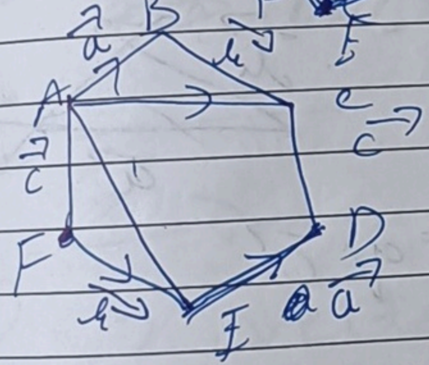


$\vec{AC} + \vec{CD} = \vec{AD}$

$\vec{a} + \vec{b} + \vec{c} = \vec{AD}$

$\vec{AE} = -\vec{b} + \vec{c}$

$\vec{AF} = -\vec{c}$



Parallel or collinear vector

2 vectors are said to be parallel or collinear if they are in same direction or opposite direction.

If $\vec{a} \parallel \vec{b}$ then $\boxed{\vec{a} = \lambda \vec{b}}$ $\lambda \in R$

$\lambda > 0$, $\vec{a}$ and $\vec{b}$ are in same direction

$\lambda < 0$, $\vec{a}$ and $\vec{b}$ are opposite "

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

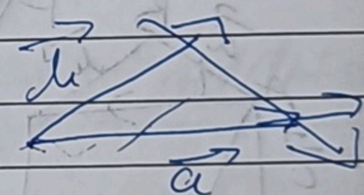
$$\boxed{\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}}$$

<u>Coplanar vectors</u>

If 3 or more vectors are in same plane then they are said to be Coplanar.

☆ If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then

☆ $\boxed{\vec{a} = x\vec{b} + y\vec{c}}$



$$\vec{AB} = p\vec{a}, \quad \vec{BC} = q\vec{c}$$

$$\vec{AC} = r\vec{b}$$

$$p\vec{a} + q\vec{c} = r\vec{b}$$

☆ $\boxed{\vec{a} = x\vec{b} + y\vec{c}}$

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

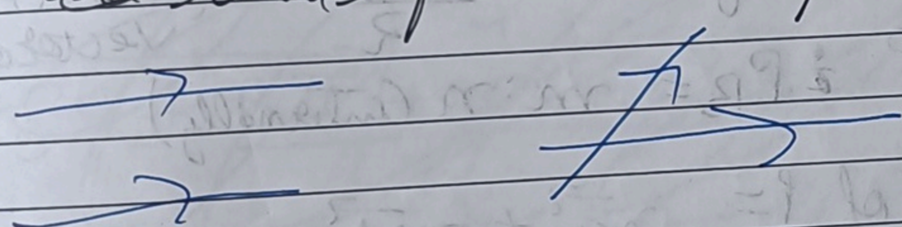
$$\boxed{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0 \quad \text{Coplanar}}$$

$$(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) = x(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) + y(c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$$

$$\begin{vmatrix} x b_1 + y c_1 & b_2 x + c_2 y & b_3 x + c_3 y \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad R_1 \to R_1 - R_2 x - R_3 y = 0$$

$$\therefore \Delta = 0$$

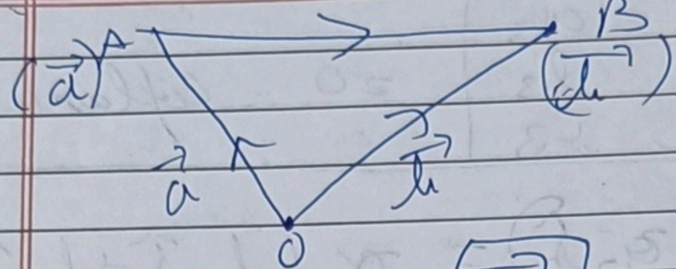
(1.) Note 2 non collinear vectors ~~are~~ will represent a plane.



(2.) 2 vectors are always coplanar or 3 points are always coplanar.

Position vector of a point

If point P is in space and O is a point taken as origin then $\vec{OP}$ is known as Position vector of P with refrence O.



$$\vec{AB}+\vec{OA}=\vec{OB}$$

$$\vec{AB}=\vec{OB}-\vec{OA}$$

$$\boxed{\vec{AB}=\text{P.V of } B-\text{P.V of } A}$$

$$\boxed{\vec{AB}=\text{P.V of } B-\text{P.V of } A}$$

Distance formula

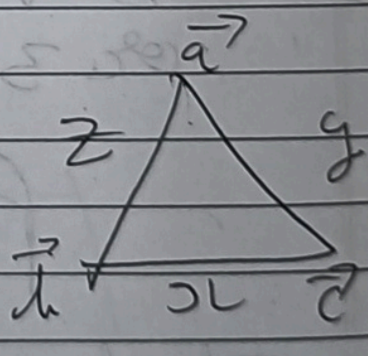
☆ $AB=|\vec{b}-\vec{a}|$ $\quad A(\vec{a}) \quad B(\vec{b})$

☆ Mid point of $\vec{AB}=\dfrac{\vec{a}+\vec{b}}{2}$ (Position vector of mid point)

☆ $AP : PB = m : n$ (internally)

$$\text{P.V of } P=\frac{n\vec{a}+m\vec{b}}{m+n}$$

☆ Centroid $=\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$



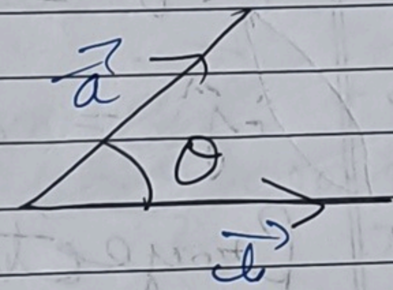
☆ Incentre $=\dfrac{x\vec{a}+y\vec{b}+z\vec{c}}{x+y+z}$

$$x=|\vec{b}-\vec{c}|,\quad y=|\vec{c}-\vec{a}|$$

$$z=|\vec{a}-\vec{b}|$$

Dot Product (Scalar Product)

$\vec{a}\cdot\vec{b} = |a|\,|b|\cos\theta$



☆ If θ is acute
$\vec{a}\cdot\vec{b} > 0$

☆ If θ is obtuse $\vec{a}\cdot\vec{b} < 0$

☆ If θ is Right Angle $\vec{a}\cdot\vec{b} = 0$

☆ $\boxed{\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|a|\,|b|}}$

☆ $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (Commutative)

☆ $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

Cross Product (vector Product)

$\vec{a} \times \vec{b} = |a||b| \sin\theta \, \hat{n}$

$\hat{n}$ is a unit vector $\perp$ to plane of $\vec{a}$ and $\vec{b}$

1) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

2) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

3) $\vec{a} \times \vec{a} = 0$

4) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

5) $\vec{a} \times \vec{b} = 0$ $\vec{a}$ and $\vec{b}$ are collinear or Parallel.

or $|\vec{a}| = 0$, $|\vec{b}| = 0$

6) $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$

$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

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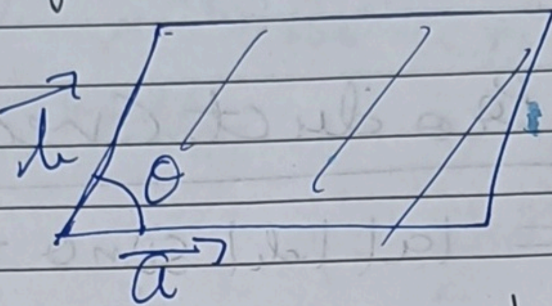
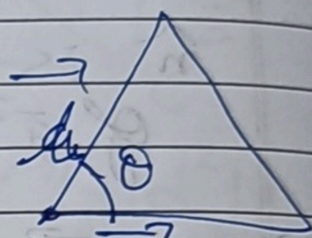
(7) $|\vec{a} \times \vec{b}|$ is area of ∥gm whose adjacent side are $\vec{a}$ and $\vec{b}$.



(8) Area of $\Delta = \frac{|\vec{a} \times \vec{b}|}{2}$



Q. Prove $(\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = |a|^2|b|^2$

$$= |a|^2|b|^2 \cos^2\theta + |a|^2|b|^2 \sin^2\theta$$
$$= |a|^2|b|^2 (\cos^2\theta + \sin^2\theta)$$
$$= |a|^2 |b|^2$$

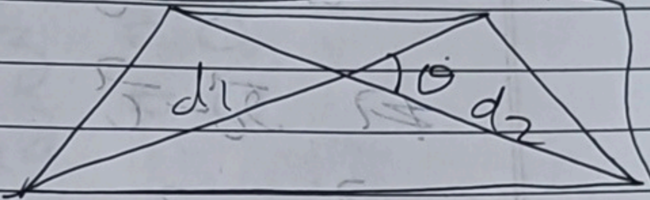
$$\boxed{\text{Result} = (\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = |a|^2|b|^2}$$

Q. Prov $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2\vec{a}^2$

~~$a^2 \sin\theta_1 + a^2 \sin\theta_2 + a^2 \sin\theta_3$~~

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$= |-a_3\hat{k} + a_2\hat{j}|$$
$$= a_3^2 + a_2^2 + a_1^2 + a_3^2 + a_1^2 + a_2^2$$
$$= 2|\vec{a}|^2$$

$$\boxed{A = \frac{1}{2} d_1 d_2 \sin\theta}$$



$$= \frac{1}{2} \left| \vec{d_1} \times \vec{d_2} \right|$$

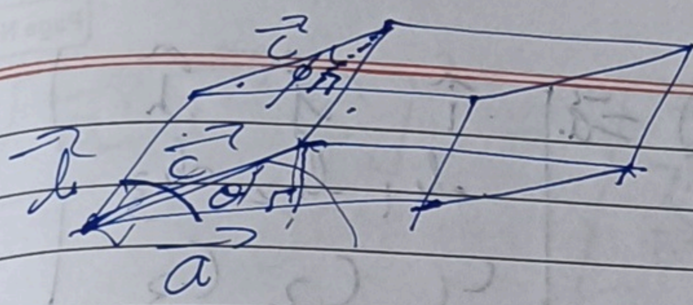
Scalar Triple Product (STP)

☆ $\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{b}\times\vec{c})\cdot\vec{a}$

$\vec{b}\cdot(\vec{c}\times\vec{a}) = (\vec{c}\times\vec{a})\cdot\vec{b}$

$\vec{c}\,(\vec{a}\times\vec{b}) = (\vec{a}\times\vec{b})\cdot\vec{c}$

☆ value of STP Volume of Parallelopiped having $\vec{a},\vec{b},\vec{c}$ as coterminal edges.



Volume = Area of base × hieght

$= |\vec{a} \times \vec{b}| \times$ hieght.

$= |a|\,|b| \sin\theta \;\; |\vec{C}| \cos\phi$

$= |a|\,|b|\,|c| \sin\theta \cos\phi$

$V = (\vec{a} \times \vec{b}) \cdot \vec{c}$

$= (|a|\,|b| \sin\theta\, \hat{n}) \cdot \vec{C}$

$= |a|\,|b| \sin\theta\, |c| \cos\phi\, |\hat{n}|$

$$\boxed{\text{Volume} = |a|\,|b|\,|c| \sin\theta \cos\phi}$$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$[\vec{a}\;\vec{b}\;\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{a}\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

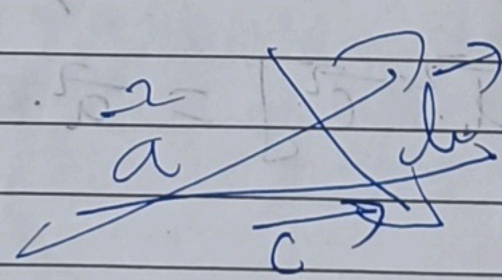
$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$[\vec{b}\ \vec{c}\ \vec{a}] = \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

$$\therefore \boxed{\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}$$

★ When any of two vectors out of three vectors are // or collinear or identical Value of STP is zero

★ If 3 vectors are coplanar value of STP is zero.



$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{bmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{bmatrix}$$

(By multiplication of determinant)

$$\vec{p} = p_1\vec{a} + p_2\vec{b} + p_3\vec{c}$$

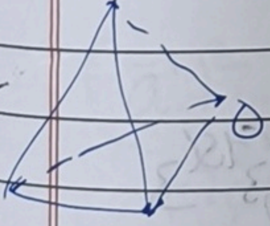
$$\vec{q} = q_1\vec{a} + q_2\vec{b} + q_3\vec{c}$$

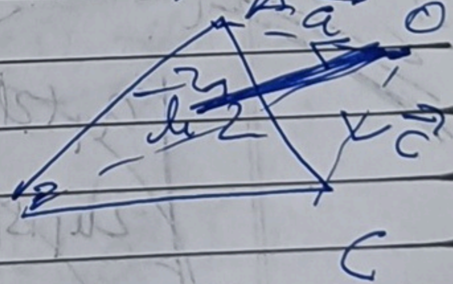
$$\vec{r} = r_1\vec{a} + r_2\vec{b} + r_3\vec{c}$$

$$\boxed{[\vec{p}\ \vec{q}\ \vec{r}] = \begin{vmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{vmatrix} \times [\vec{a}\ \vec{b}\ \vec{c}]}$$

Volume of tetrahedron

$$= \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$$





Volume of tetrahedron = $\boxed{\frac{1}{3}\text{(area of base)} \times \text{height}}$

$$\boxed{V = \frac{1}{3}\text{(area of base)} \times \text{hiegtd}}$$