PROPERTIES OF A.P. :

(a) If each term of an A.P. is increased, decreased, multiplied or divided by the some nonzero number, then the resulting sequence is also an A.P.

(b) Three numbers in A.P. : $a - d, a, a + d$

Four numbers in A.P. : $a - 3d, a - d, a + d, a + 3d$

Five numbers in A.P. : $a - 2d, a - d, a, a + d, a + 2d$

Six numbers in A.P. : $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.

(c) The common difference can be zero, positive or negative.

(d) k^{th} term from the last = $(n - k + 1)^{th}$ term from the beginning (If total number of terms = n).

(e) The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms. $\Rightarrow T_k + T_{n-k+1} = \text{constant} = a + \ell$.

(f) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it. $a_n = (1/2)(a_{n-k} + a_{n+k}),\ k < n$

For $k = 1,\ a_n = (1/2)(a_{n-1} + a_{n+1})$; For $k = 2,\ a_n = (1/2)(a_{n-2} + a_{n+2})$ and so on.

(g) If a, b, c are in AP, then $2b = a + c$.

ARITHMETIC MEAN :

If three terms are in A.P. then the middle term is called the A.M. between the other two, so if a, b, c are in A.P., b is A.M. of a & c. So A.M. of a and c = $\frac{a+c}{2} = b$.

n-ARITHMETIC MEANS BETWEEN TWO NUMBERS :

If a,b be any two given numbers & a, A_1, A_2, ..., A_n, b are in AP, then A_1, A_2,........A_n are the 'n' A.M's between a & b then. $A_1 = a + d$, $A_2 = a + 2d$,......, $A_n = a + nd$ or $b - d$, where

$$d = \frac{b-a}{n+1}$$

$$\Rightarrow A_1 = a + \frac{b-a}{n+1}, \quad A_2 = a + \frac{2(b-a)}{n+1}, \ldots\ldots$$

Note : Sum of n A.M's inserted between a & b is equal to n times the single A.M. between a & b i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single A.M. between a & b.

GEOMETRIC PROGRESSION (G.P.) :

G.P. is a sequence of non zero numbers each of the succeeding term is equal to the preceeding term multiplied by a constant. Thus in a GP the ratio of successive terms is constant. This constant factor is called the COMMON RATIO of the sequence & is obtained by dividing any term by the immediately previous term. Therefore a, ar, ar^2, ar^3, ar^4 , ... is a GP with 'a' as the first term & 'r' as common ratio.

(a) n^{th} term ; $T_n = a\, r^{n-1}$

(b) Sum of the first n terms; $S_n = \dfrac{a(r^n - 1)}{r - 1}$, if $r \neq 1$

(c) Sum of infinite G.P. , $S_\infty = \dfrac{a}{1-r}$; $0 < |r| < 1$

PROPERTIES OF GP :

(a) If each term of a G.P. be multiplied or divided by the some non-zero quantity, then the resulting sequence is also a G.P.

(b) Three consecutive terms of a GP : a/r, a, ar ;
Four consecutive terms of a GP : a/r^3, a/r, ar, ar^3 & so on.

(c) If a, b, c are in G.P. then $b^2 = ac$.

(d) If in a G.P, the product of two terms which are equidistant from the first and the last term, is constant and is equal to the product of first and last term. $\Rightarrow T_k . T_{n-k+1} = \text{constant} = a.\ell$

(e) If each term of a G.P. be raised to the same power, then resulting sequence is also a G.P.

(f) In a G.P., $T_r^2 = T_{r-k} . T_{r+k}$, $k < r$, $r \neq 1$

(g) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(h) If $a_1, a_2, a_3 \ldots\ldots a_n$ is a G.P. of positive terms, then $\log a_1, \log a_2, \ldots\ldots \log a_n$ is an A.P. and vice-versa.

(i) If $a_1, a_2, a_3 \ldots\ldots$ and $b_1, b_2, b_3 \ldots\ldots$ are two G.P.'s then $a_1b_1, a_2b_2, a_3b_3 \ldots\ldots$ & $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3} \ldots\ldots\ldots\ldots$ is also in G.P.