DEFINITION :

Sequence :

A succession of terms a_1, a_2, a_3, a_4....... formed according to some rule or law.

Examples are : $1, 4, 9, 16, 25$

$-1, 1, -1, 1, \ldots\ldots$

$\frac{x}{1!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, \ldots\ldots$

A finite sequence has a finite (i.e. limited) number of terms, as in the first example above. An infinite sequence has an unlimited number of terms, i.e. there is no last term, as in the second and third examples.

Series :

The indicated sum of the terms of a sequence. In the case of a finite sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ the corresponding series is $a_1 + a_2 + a_3 + \ldots\ldots + a_n = \sum_{k=1}^{n} a_k$. This series has a finite or limited number of terms and is called a finite series.

ARITHMETIC PROGRESSION (A.P.) :

A.P. is a sequence whose terms differ by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then A.P. can be written as

$$a,\ a + d,\ a + 2d,\ \ldots\ldots\ldots\ldots\ldots,\ a + (n-1)\,d\ ,\ \ldots\ldots\ldots$$

(a) n^{th} term of AP $T_n = a + (n-1)d$, where $d = t_n - t_{n-1}$

(b) The sum of the first n terms : $S_n = \frac{n}{2}[a + \ell] = \frac{n}{2}[2a + (n-1)d]$

where ℓ is n^{th} term.

Note :

(i) n^{th} term of an A.P. is of the form $An + B$ i.e. a linear expression in 'n', in such a case the coefficient of n is the common difference of the A.P. i.e. A.

(ii) Sum of first 'n' terms of an A.P. is of the form $An^2 + Bn$ i.e. a quadratic expression in 'n', in such case the common difference is twice the coefficient of n^2. i.e. 2A

(iii) Also n^{th} term $T_n = S_n - S_{n-1}$

SIGMA NOTATIONS (Σ)

THEOREMS :

(a) $\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$ **(b)** $\sum_{r=1}^{n} k\,a_r = k\sum_{r=1}^{n} a_r$ **(c)** $\sum_{r=1}^{n} k = nk$; where k is a constant.

RESULTS

(a) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ (sum of the first n natural numbers)

(b) $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(c) $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2$ (sum of the cubes of the first n natural numbers)

(d) $\sum_{r=1}^{n} r^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$

(e) $\sum_{r=1}^{n}(2r-1) = n^2$ (sum of first n odd natural numbers)

(f) $\sum_{r=1}^{n} 2r = n(n+1)$ (sum of first n even natural numbers)

Note :

If n^{th} term of a sequence is given by $T_n = an^3 + bn^2 + cn + d$ where a, b, c, d are constants,

then sum of n terms $S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d$

This can be evaluated using the above results.