JEE Main Maths Limits, Continuity and Differentiability Previous Year Questions With Solutions

Question 1: Solve

$$\lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$$

Solution:

$$\lim_{x \to 1} \frac{(2x-3)\,(\sqrt{x}-1) \times (\sqrt{x}+1)}{(x-1)\,(2x+3) \times (\sqrt{x}+1)}$$
$$= \frac{-1}{5\,.\,2}$$
$$= \frac{-1}{10}$$

Question 2: If f(9)=9, f'(9)=4, then

$$\lim_{x \to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$$

Solution:

Applying L – Hospitals rule,

$$\lim_{x \to 9} \frac{\frac{1}{2\sqrt{f(x)}} \cdot f'(x)}{\frac{1}{2\sqrt{x}}}$$
$$= \frac{\frac{f'(9)}{\sqrt{f(9)}}}{\frac{1}{\sqrt{9}}}$$
$$= \frac{\frac{4}{3}}{\frac{1}{3}}$$
$$= 4$$

Question 3: Solve

$$\lim_{h \to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$$

Solution:

Apply L-Hospitals rule,

$$\lim_{h \to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$$

$$\lim_{h \to 0} \frac{2(a+h)\ \sin(a+h)+(a+h)^2\cos(a+h)}{1}$$
$$= 2a\ \sin a + a^2 \cos\ a$$

Question 4: Solve

$$\lim_{x \to \pi/4} \frac{\sqrt{2}\cos x-1}{\cot x-1}$$

Solution:

$$\lim_{x \to \pi/4} \frac{(\sqrt{2}-\sec x)\ \cos x\ (1+\cot x)}{\cot x\ [2-\sec^2 x]}$$
$$= \lim_{x \to \pi/4} \frac{\sin x\ (1+\cot x)}{(\sqrt{2}+\sec x)}$$
$$= \frac{\frac{1}{\sqrt{2}}(2)}{\sqrt{2}+\sqrt{2}}$$
$$= \frac{1}{2}$$

Question 5: Solve

$$\lim_{x \to 0} \left[\frac{x}{\tan^{-1}2x}\right]$$

Solution:

Let

$$\tan^{-1}2x = \theta$$
$$\Rightarrow x = \frac{1}{2}\tan\theta\ and\ as\ x \to 0,\ \theta \to 0$$
$$\Rightarrow\ \lim_{x \to 0} \frac{x}{\tan^{-1}2x}$$
$$= \lim_{\theta \to 0} \frac{\frac{1}{2}\tan\theta}{\theta}$$
$$= \frac{1}{2}$$

Question 6: Solve

$$\lim_{x \to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$$

Solution:

$$\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x} = \lim_{x\to 0} \frac{|\sin x|}{x}$$

So,

$$\lim_{x\to 0+} \frac{|\sin x|}{x} = 1$$

and

$$\lim_{x\to 0-} \frac{|\sin x|}{x} = -1$$

Hence, the limit doesn't exist.

Question 7: Solve

$$\lim_{x\to 0} \left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{1/x}$$

Solution:

Given limit =

$$= \lim_{x\to 0} \left(\frac{1+\tan x}{1-\tan x}\right)^{1/x}$$

$$= \lim_{x\to 0} \frac{\left\{(1+\tan x)^{1/\tan x}\right\}^{(\tan x)/x}}{\left\{(1-\tan x)^{1/\tan x}\right\}^{(\tan x)/x}}$$

$$= \frac{e}{e^{-1}}$$

$$= e^2.$$

Question 8: Solve

$$\lim_{x\to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$$

Solution:

$$\lim_{x\to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$$

$$= \frac{\lim_{x\to 0} \left[(1+5x^2)^{1/5x^2}\right]^5}{\lim_{x\to 0} \left[(1+3x^2)^{1/3x^2}\right]^3}$$

$$= \frac{e^5}{e^3}$$

$$= e^2$$

$= e^2$

$[\because \ \lim_{x \to 0} \ (1+x)^{1/x} = e]$

Question 9: Solve

$\lim_{x \to 0} \ \frac{x \tan 2x - 2x \tan x}{(1-\cos 2x)^2}$

Solution:

$\lim_{x \to 0} \ \frac{x \tan 2x - 2x \tan x}{(1-\cos \ 2x)^2}$
$= \lim_{x \to 0} \ \frac{x(\tan \ 2x - 2\tan x)}{(2\sin^2 x)^2}$
$= \lim_{x \to 0} \ \frac{1}{4} \ \frac{x\,(\tan 2x - 2\tan x)}{\sin^4 x}$
$= \lim_{x \to 0} \ \frac{1}{4} \frac{x\left\{\left(2x + \frac{1}{3}(2x)^3 + \frac{2}{15}\ (2x)^5 + \ldots\right) - 2\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\right)\right\}}{x^4\left(1 - \frac{x^2}{3\,!} + \frac{x^4}{5\,!} + \ldots\right)^4}$
$= \frac{1}{4} \cdot \left(\frac{8}{3} - \frac{2}{3}\right)$
$= \frac{2}{4}$
$= \frac{1}{2}.$

Question 10: The function

$f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$
is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0, is

Solution:

Since limit of a function is a + b as x → 0, therefore to be continuous at a function, its value must be a + b at x = 0

⇒ f (0) = a + b

Question 11: Evaluate

$$f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$$

Solution:

Solution:

For continuous

$\lim_{x \to 2}\ f(x) = f(2) = k$

$\Rightarrow\ k = \lim_{x \to 2} \frac{x^3+x^2-16x+20}{(x-2)^2}$

$= \lim_{x \to 2}\ \frac{(x^2-4x+4)\ (x+5)}{(x-2)^2}$

$= 7.$

Question 12:

$f(x) = \{\ \begin{matrix} \frac{x^2-4x+3}{x^2-1} & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{matrix}$

, then find the condition for the function to be continuous or discontinuous.

Solution:

$f(x) = \left\{ \frac{x^2-4x+3}{x^2-1} \right\}$

,

for x = 1

f (1) = 2,

$f(1+) = \lim_{x \to 1+}\ \frac{x^2-4x+3}{x^2-1}$

$= \lim_{x \to 1+}\ \frac{(x-3)}{(x+1)}$

$= -1$

$f(1-) = \lim_{x \to 1-}\ \frac{x^2-4x+3}{x^2-1}$

$= -1$

$\Rightarrow\ f(1) \neq f(1-)$

Hence, the function is discontinuous at x = 1.

Question 13: Which of the following functions have a finite number of points of discontinuity in R ([.] represents the greatest integer function)?

A) tanx

Question 13: Which of the following functions have a finite number of points of discontinuity in R ([.] represents the greatest integer function)?

A) tanx

B) x[x]

C) |x| / x

D) sin[πx]

Solution:

f (x) = tanx is discontinuous when x = (2n + 1) π / 2, n ∈ Z

f (x) = x[x] is discontinuous when x = k, k ∈ Z

f (x) = sin [nπx] is discontinuous when nπx = k, k ∈ Z

Thus, all the above functions have an infinite number of points of discontinuity. But, if (x) = |x| / x is discontinuous when x = 0 only.

Question 14: The number of values of x ∈ [0, 2] at which f (x) = |x − [1 / 2]| + |x − 1|+ tanx is not differentiable is

A) 0

B) 1

C) 3

D) None of these

Solution:

|x − [1 / 2]| is continuous everywhere but not differentiable at x = 1 / 2, |x − 1| is continuous everywhere, but not differentiable at x = 1 and tan x is continuous in [0, 2] except at x = π / 2. Hence, f (x) is not differentiable at x = 1 / 2, 1, π / 2.

Question 15:

$\lim_{x \to \pi/2} (\sec\theta - \tan\theta) =$

Solution:

$$\lim_{\theta \to \pi/2} \frac{1-\sin\theta}{\cos\theta} = \lim_{\theta \to \pi/2} \frac{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)} = 0$$