

① mass of a box = 2.3 kg  
Two marbles of mass  
2.15g and 12.39g are placed  
in the box.  
Total mass of the box correct  
up to the number of significant  
digits.

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Total mass of box + marbles

$$2.3 \text{ kg} + 0.00215 \text{ kg} \\ + 0.01239 \text{ kg}$$

$$= 2.31454 \text{ kg}$$

$$= 2.3 \text{ kg}$$

Total mass of box + marbles

$$\begin{aligned} & 2.3 \text{ kg} + 0.00215 \text{ kg} \\ & \quad \quad \quad + 0.01239 \text{ kg} \\ & = 2.31454 \text{ kg} \\ & = 2.3 \text{ kg} \end{aligned}$$

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mass of box = 2.300 kg  
up to 3 decimal      ↑  
2.315 kg

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Two lengths  $L_1$  and  $L_2$  are measured.

$$L_1 = 9.99 \text{ m}, \quad L_2 = 9.9 \text{ mm}$$

Sum = ?

$$L_1 = 9.99 \text{ m}, \quad L_2 = 0.0099 \text{ m}$$

$$L_1 + L_2 = 9.9999 \text{ m}$$

Correct up to 2 decimal places

$$= 10.00 \text{ m}$$

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Each side of a cube  
= 5402 cm

Find surface area of cube in  
appropriate significant figures.

$$S \text{ Area} = 6a^2$$

# of significant digits = 4

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$$6a^2 = 175.089624 \text{ cm}^2$$

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To determine  $g$  use formula

$$T = 2\pi \sqrt{\frac{7(R-r)}{5g}}$$

$R \rightarrow 60 \pm 1 \text{ mm}$   
 $r \rightarrow 10 \pm 1 \text{ mm}$

5 experiments:

$T$  measured as 0.52s, 0.56s, 0.57s  
0.54s and 0.59s

Least count of watch = 0.01s

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Find % error in measurement of  $\tau$ ,  $T$  and  $g$ .

$$T_{\text{mean}} = \frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5}$$
$$= 0.556 = 0.56 \text{ s}$$

$$|\Delta T_{\text{mean}}| = \frac{1}{5} (|\Delta T_1| + |\Delta T_2| + \dots + |\Delta T_n|)$$
$$|\Delta T_1| = |0.52 - 0.56| = 0.04$$

$$|\Delta T_5| = |0.59 - 0.56| = 0.03$$

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$$\begin{aligned}\text{Mean error} &= \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.03}{5} \\ &= \frac{0.1}{5} = 0.02 \text{ s}\end{aligned}$$

$$\begin{aligned}\% \text{ error in } T &= \frac{\Delta T}{T} \times 100 \\ &= \frac{0.02}{0.56} \times 100 = 3.57\%\end{aligned}$$


$$\% \text{ error in } r = \frac{\Delta r}{r} \times 100 = \frac{1}{10} \times 100 = 10\%$$



$$g = \frac{28\pi^2}{5} \left( \frac{R-r}{T^2} \right)$$
$$\frac{\Delta g}{g} = \frac{\Delta(R-r)}{R-r} + 2 \frac{\Delta T}{T}$$
$$\Delta(R-r) = \Delta R + \Delta r = 1 + 1 = 2 \text{ mm}$$
$$(R-r) = 60 - 10 = 50 \text{ mm}$$
$$\frac{\Delta g}{g} = \frac{2}{50} + \frac{2 \times 0.02}{0.56}$$

% can be obtained by multiplying by 100



$$\frac{\Delta g}{g} \times 100 = \frac{200}{50} + \frac{2 \times 0.02}{0.56} \times 100$$
$$= 4 + 7.14 = 11.14\%$$


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All quantities can be expressed  
in 7 basic dimensions

- |       |   |             |                        |
|-------|---|-------------|------------------------|
| $[L]$ | → | Length      | Lumin Intensity $[cd]$ |
| $[M]$ | → | mass        | moles $[mol]$          |
| $[T]$ | → | Time        |                        |
| $[A]$ | → | Current     |                        |
| $[K]$ | → | Temperature |                        |

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Dimensional consistency:  
quantities of same dimensions  
can be added or subtracted.

cannot add Force and velocity

$$[\text{speed}] = \frac{\text{Dist}}{\text{Time}} \quad \left[ \frac{L}{T} \right] = [LT^{-1}]$$

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Force:  $\sim ma$

$$[F] = [M] \left[ \frac{L}{T^2} \right] = [MLT^{-2}]$$

Principle of Dimensional Homogeneity  
all terms in some equation which  
are added or subtracted have  
same dimensions.

$A = B$





Dimensional consistency is  
necessary  $[A] = [B]$   
if  $A = B$

Dimensional consistency does not  
ensure that formula is correct  
but if dimensional consistency is  
absent, formula/equation is  
incorrect

Certain quantities which are dimensionless.

Egs.

- 1) angles
- 2) ratios of similar physical quantities  
eg. refractive index

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e.g. Time period  $T$  of vibration of a drop depends on surface tension  $S$ , radius and density  $\rho$  of liquid.

Find an expression for  $T$

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$$T \propto S^\alpha r^\beta \rho^\gamma$$

$\alpha, \beta, \gamma$  are unknowns

$$[T] = [L^0 M^0 T^1]$$
$$[S] = \frac{\text{Force}}{\text{Length}} = \left[ M \frac{L}{T^2} \cdot \frac{1}{L} \right] = [L^0 M T^{-2}]$$
$$[r] = [L M^0 T^0] = [L]$$
$$[\rho] = \frac{\text{mass}}{\text{Volume}}$$

$\alpha, \beta, \gamma$  are unknowns

$$[T] = [L^0 M^0 T^1]$$

$$[S] = \frac{\text{Force}}{\text{Length}} = \left[ M \frac{L}{T^2} \cdot \frac{1}{L} \right] = [L^0 M T^{-2}]$$

$$[\gamma] = [L M^0 T^0] = [L]$$

$$[\rho] = \frac{\text{mass}}{\text{Volume}} = \frac{m}{L^3} = [M L^{-3}]$$

$$[L^0 M^0 T^1] = [M T^{-2}]^{\alpha} [L]^{\beta} [ML^{-3}]^{\gamma}$$

↑                    ↑                    ↑  
α                    β                    γ

equate powers of L, M, T separately

$$L: 0 = \beta - 3\gamma$$

$$M: 0 = \alpha + \gamma$$

$$T: 1 = -2\alpha \quad \leftarrow \alpha = -\frac{1}{2}$$

$$\gamma = \frac{1}{2}, \quad \beta = \frac{3}{2}$$

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$$T \propto S^\alpha r^\beta \rho^\gamma$$
$$T = K S^{-1/2} r^{3/2} \rho^{1/2}$$
$$= K \sqrt{\frac{r^3 \rho}{S}}$$

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Ex. Potential energy of a particle varies with distance  $x$  from origin

as: 
$$U = \frac{A\sqrt{x}}{x^2 + B}$$

$A$  and  $B$  are dimensional constants.  
Find the dimensional formula for  $AB$ .

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$[AB]$ :  $\rightarrow$  find dimensions of A  
dimensions of B.

$x^2 + B \Rightarrow$  dimension of B  
 $=$  dimension of  $x^2$

$$[B] = [L^2]$$

$$A = \frac{v(x^2 + B)}{\sqrt{x}}$$

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$$A = \frac{U(x^2+B)}{\sqrt{x}}$$
$$[A] = \frac{[U] [x^2]}{[\sqrt{x}]}$$
$$[U] = \text{Energy} \rightarrow [m v^2]$$
$$= [M L^2 T^{-2}]$$

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$$\begin{aligned} [A] &= [M L^2 T^{-2}] [L^2] [L^{-1/2}] \\ [A] &= [M L^{7/2} T^{-2}] \\ [AB] &= [M L^{7/2} T^{-2}] [L^2] \\ &= [M L^{11/2} T^{-2}] \end{aligned}$$

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Limitations

1. 2 Physical quantities which are not related can have same dimensions.

e.g. a) Moment of a force or Torque  
 $[F][d]$

b) Kinetic energy or Work done  
 $[F][d]$

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2) formulae correct up to a constant  $K$ , i.e. dimensionless.

Dimensional analysis cannot help in finding the value of  $K$ .

3) Cannot use dimensional analysis to predict behaviour where eqns. have quantities other than powers of products.

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$$e.g. \quad y = y_0 \sin(\omega t)$$

$$s = \underbrace{ut} + \frac{1}{2} \underbrace{at^2}$$

d) Limited form of  
π theorem of Buckingham

frequency  
of  
tuning fork

$$f = \frac{d}{L^2} \sqrt{V}$$

formula cannot  
be derived  
from  
dimensional  
analysis

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