Q.1. A single electron orbits around a stationary nucleus of charge + Ze. Where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit.

Find (1981- 10 Marks)

(i) The value of Z.
(ii) The energy required to excite the electron from the third to the fourth Bohr orbit.
(iii) The wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.
(iv) The kinetic energy, potential energy, potential energy and the angular momentum of the electron in the first Bohr orbit.
(v) The radius of the first Bohr orbit.

(The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} metre, velocity of light = 3×10^{8} m/sec.

Planck's constant = 6.6×10^{-34} joules - sec).

Solution.

$$E_2 = -\frac{13.6}{4}Z^2,\ E_3 = -\frac{13.6}{9}Z^2$$

$$E_3 - E_2 = -13.6Z^2\left(\frac{1}{9}-\frac{1}{4}\right) = +\frac{13.6\times 5}{36}Z^2$$

But $E_3 - E_2 = 47.2$ eV (Given)

$$\therefore \frac{13.6\times 5}{36}Z^2 = 47.2 \quad \therefore Z = \frac{\sqrt{47.2\times 36}}{13.6\times 5} = 5$$

(ii) $E_4 = \frac{-13.6}{16}Z^2$

$$\therefore E_4 - E_3 = -13.6Z^2\left[\frac{1}{16}-\frac{1}{9}\right] = -13.6Z^2\left[\frac{9-16}{9\times 16}\right]$$

$$= \frac{+13.6\times 25\times 7}{9\times 16} = 16.53\,\text{eV}$$

(iii) $E_1 = -\frac{13.6}{1}\times 25 = -340\,\text{eV}$

$\therefore E = E_\infty - E_1 = 340\,\text{eV} = 340\times 1.6\times 10^{-19}\,J \quad [E_\infty = 0\,\text{eV}]$

But $E = \frac{hc}{\lambda}$

$$\therefore \lambda = \frac{hc}{E} = \frac{6.6\times 10^{-34}\times 3\times 10^8}{340\times 10^{-19}\times 1.6} = 3.65\times 10^{-19}\,\text{m}$$

(iv) Total Energy of 1st orbit = – 340 eV We know that – (T.E). = K.E. [in case of electron revolving around nucleus] and 2T.E. = P.E.

∴ K.E. = 340 eV ; P.E. = – 680 eV

KEY CONCEPT :

Angular momentum in 1st orbit :

According to Bohr's postulate,

$$mvr = \frac{nh}{2\pi}$$

For $n = 1$,

$$mvr = \frac{h}{2\pi} = \frac{6.6\times 10^{-34}}{2\pi} = 1.05\times 10^{-34}\,\text{J-s.}$$

(v) Radius of first Bohr orbit

$$r_1 = \frac{5.3\times 10^{-11}}{Z} = \frac{5.3\times 10^{-11}}{5}$$

$= 1.06 \times 10^{-11}$ m