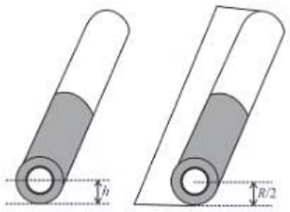
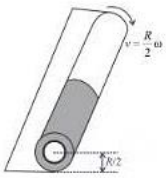
Ques.

A carpet of mass *M* and of inextensible material is rolled along its length in the form of a cylinder of radius *R* and is kept on a rough floor. The carpet starts unrolling without sliding on the floor where a negligibly small push is given to it. The horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to *R*/2. is $\sqrt{\frac{xgR}{3}}ms^{-1}$, then *x* is :





$M_{\text{initial}} = \pi R^2 L \rho$

$M_{\text{final}} = \left(\frac{R}{2}\right)^2 \cdot L \cdot \rho = \frac{M_i}{4}$

Initial PE of carpet $= Mg \cdot R$

Final PE of carpet $= \frac{M}{4} \cdot g \cdot \frac{R}{2} = \frac{MgR}{8}$

ΔPE decrease $= \frac{7}{8} MgR$

It's equal to gain in KE.

$= K_{\text{tran}} + K_{\text{Rot}} = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$

using mass $= \frac{M}{4}$

$v = \frac{R}{2} \cdot \omega \, I = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$

$K = \frac{1}{2}\left(\frac{M}{4}\right) v^2 + \frac{1}{2}\left(\frac{MR^2}{32}\right)\left(\frac{2v}{R}\right)^2 = \frac{3}{16} Mv^2$

Equating $\frac{7}{8} MgR = \frac{3}{16} Mv^2$

$v = \sqrt{\frac{14gR}{3}}$

The correct answer is: 14