Vectors Formulas

Basic Formulas

$\vec{A}\ (x_1, x_2, x_3)$ $\vec{B}(y_1, y_2, y_3)$

Components

$$\vec{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

Magnitude or Length

$$\vec{u} = (u_1, u_2, u_3)$$

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

Distance between two points

$$d(A, B) = \left|\vec{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Unit Vector

$$\vec{u}_{\vec{v}} = \frac{\vec{v}}{|\vec{v}|}$$

Vector Addition

$$\vec{u} = (u_1, u_2, u_3) \qquad \vec{v} = (v_1, v_2, v_3)$$

$$\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$$

Scalar Multiplication

$$k.\vec{u} = (ku_1 + ku_2 + ku_3)$$

Linearly Dependent Vectors

$$\vec{u} = k\vec{v} \qquad (u_1, u_2, u_3) = (kv_1 + kv_2 + kv_3)$$

$$\frac{u_1}{v_1} = \frac{u_2}{v_2} = \frac{u_3}{v_3} = k$$

$$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = 0$$

Linearly Independent Vectors

$$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \neq 0$$

Dot Product

$$\vec{u}.\vec{v} = |\vec{u}| \,.\, |\vec{v}| \,.\cos\alpha$$

$$\vec{u}.\vec{v} = u_1.v_1 + u_2.v_2 + u_3.v_3$$

Magnitude of a Vector

$$|\vec{u}| = \sqrt{\vec{u}.\vec{u}} = \sqrt{u_1.u_1 + u_2.u_2 + u_3.u_3} = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

Angle Between Two Vectors

$$\cos\alpha = \frac{u_1.v_1 + u_2.v_2 + u_3.v_3}{\sqrt{u_1^2 + u_2^2 + u_3^2}.\sqrt{v_1^2 + v_2^2 + v_3^2}}$$

Orthogonal Vectors

$$\vec{u}.\vec{v} = (u_1.v_1 + u_2.v_2 + u_3.v_3) \times 0 = 0$$

Direction Cosine

$$\cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

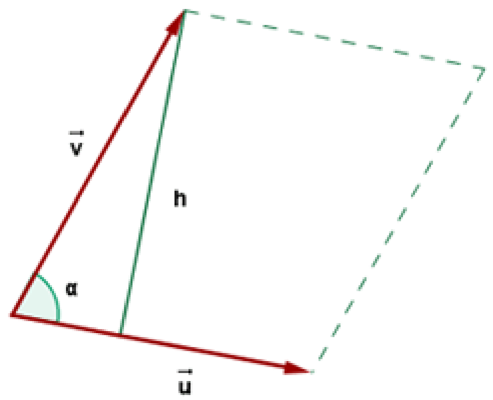
Cross Product

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \vec{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \vec{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \vec{k}$$

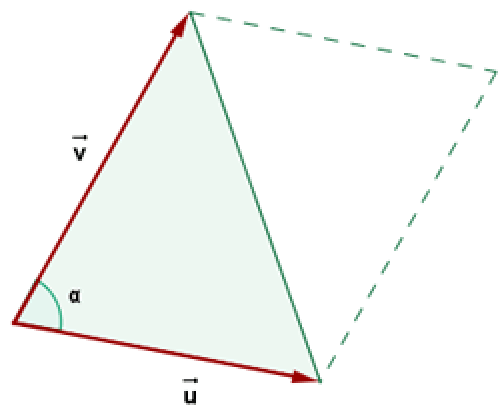
$$|\vec{u} \times \vec{v}| = |\vec{u}|\,|\vec{v}| \sin\alpha$$

Area of a Parallelogram

$$A = |\vec{u}|\,.h = |\vec{u}|\,|\vec{v}| \sin\alpha = |\vec{u} \times \vec{v}|$$



Area of a Triangle



$$A = \frac{1}{2} |\vec{u} \times \vec{v}|$$

Scalar Triple Product

$$[\vec{u}, \vec{v}, \vec{w}] = \vec{u}.(\vec{v} \times \vec{w})$$

$$[\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Volume of a Parallelepiped

$$\vec{u} = (3, -2, 5) \quad \vec{v} = (2, 2, -1) \quad \vec{w} = (-4, 3, 2)$$

Volume of a tetrahedron

$$V = \frac{1}{6} \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$