

5. Consider the system of linear equations in  $x, y, z$  :

$$(\sin 3\theta) x - y + z = 0$$

$$(\cos 2\theta) x + 4y + 3z = 0$$

$$2x + 7y + 7z = 0$$

Find the values of  $\theta$  for which this system has nontrivial solutions.

*(1986 - 5 Marks)*

5. The system will have a non-trivial solution if

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

Expanding along  $C_1$ , we get

$$\Rightarrow (28 - 21) \sin 3\theta - (-7 - 7) \cos 2\theta + 2(-3 - 4) = 0$$

$$\Rightarrow 7 \sin 3\theta + 14 \cos 2\theta - 14 = 0$$

$$\Rightarrow \sin 3\theta + 2 \cos 2\theta - 2 = 0$$

$$\Rightarrow 3 \sin \theta - 4 \sin^3 \theta + 2(1 - 2 \sin^2 \theta) - 2 = 0$$

$$\Rightarrow 4 \sin^3 \theta + 4 \sin^2 \theta - 3 \sin \theta = 0$$

$$\Rightarrow \sin \theta (2 \sin \theta - 1) (2 \sin \theta + 3) = 0$$

$$\sin \theta = 0 \text{ or } \sin \theta = 1/2 \text{ (} \sin \theta = -3/2 \text{ not possible)}$$

$$\Rightarrow \theta = n\pi \text{ or } \theta = n\pi + (-1)^n \pi/6, n \in \mathbb{Z}.$$