Q. 5 $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$

Sol. We have, $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = \begin{vmatrix} 2x+4 & 2x+4 & 2x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$ $[\because R_1 \to R_1 + R_2]$

$$= \begin{vmatrix} 2x & 2x & 2x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} + \begin{vmatrix} 4 & 4 & 0 \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

[here, given determinant is expressed in sum of two determinants]

$$= 2x \begin{vmatrix} 1 & 1 & 1 \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} + 4 \begin{vmatrix} 1 & 1 & 0 \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

[taking $2x$ common from first row of first determinant and 4 from first row of second determinant]

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ in first and applying $C_1 \to C_1 - C_2$ in second, we get

$$= 2x \begin{vmatrix} 0 & 0 & 1 \\ 0 & 4 & x \\ -4 & -4 & x+4 \end{vmatrix} + 4 \begin{vmatrix} 0 & 1 & 0 \\ -4 & x+4 & x \\ 0 & x & x+4 \end{vmatrix}$$

Expanding both the along first column, we get

$$2x[-4(-4)] + 4[4(x+4-0)]$$
$$= 2x \times 16 + 16(x+4)$$
$$= 32x + 16x + 64$$
$$= 16(3x+4)$$