Q- $A \rightarrow$ Symmetric matrix, $B \rightarrow$ Skew symmetric matrix & $(A+B)$ is non singular &

$$C = (A+B)^{-1}(A-B)$$ then which of the following option(s) is (are) correct?

A) $C^T(A+B)C = A+B$

B) $C^T(A-B)C = A-B$

C) $C^TAC = A$

D) None of these

$A^T = A, \quad B^T = -B$

Soln- $C^T(A+B)C = \{(A+B)^{-1}(A-B)\}^T (A+B)C$

$= \{(A-B)^T((A+B)^T)^{-1}\}(A+B)(A+B)^{-1}(A-B)$

$= \{(A^T - B^T)((A+B)^T)^{-1}\}(A-B)$

$= (A+B)(A^T+B^T)^{-1}(A-B)$

$= (A+B)(A-B)^{-1}(A-B)$

$\boxed{C^T(A+B)C = (A+B)} \longrightarrow (1)$

Option (A) is correct.

$C^T(A-B)C = \{(A+B)^{-1}(A-B)\}^T (A-B)(A+B)^{-1}(A+B)$

$= (A-B)^T\{(A+B)^T\}^{-1}(A-B)(A+B)^{-1}(A-B)$

$= (A^T-B^T)\{(A^T+B^T)^{-1}\}(A-B)(A+B)^{-1}(A-B)$

$= (A+B)(A-B)^{-1}(A-B)(A+B)^{-1}(A-B)$

$= (A+B)(A+B)^{-1}(A-B)$

$\boxed{C^T(A-B)C = (A-B)} \longrightarrow (2)$

Option (B) is correct.

from (1) + (2)

$2C^TAC = 2A$

$\boxed{C^TAC = A}$ Option (C) is correct.

(A, B, C) ⇒ correct.