→ A & B are square matrices of same order. If $A.B = B.A$ then $(A+B)^n$ can be expanded using Binomeal Theorem.

Hence, $(A+I)^n$ can be expanded using Binomeal theorem because $A \cdot I = I \cdot A$.

Except this, the formulas like $(a+b)^2$, a^3+b^3, a^3-b^3, a^2-b^2 etc. can also be used in such case.

<u>Ex.</u> $A^3+I = A^3+I^3$

$$= (A+I)(A^2-A+I)$$

→ If $A \cdot B = 0$

where $B \neq 0$, then $|A| = 0$

(But it is <u>not</u> <u>necessary</u> that $\underline{A=0}$)

<u>To prove</u>- Let's assume $|A| \neq 0$

then A is invertible.

$A^{-1} A B = A^{-1} \cdot 0$ {multiplying by A^{-1} both side}

$B = 0$

but it's not possible

hence $|A| \neq 0$ is not possible

So, $\boxed{|A| = 0}$ Proved

→ If A is given & some higher power (like - A^{20}, A^{50} etc.) is required then -

Method-1 We 1 calculate A^2, A^3 (sometimes A^4 also) & observing these we find out our required term.

Method-2 This method is applicable only for some special cases (i.e. 'A' is triangular matrix & all diagonal elements of 'A' are 'unity' (1).)

Ex.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

Now, we break this as $A = B + I$

where $B = \begin{bmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ b & c & 0 \end{bmatrix}$

Now, $B^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ ac & 0 & 0 \end{bmatrix}$

$B^n = O$ for $n \geq 3$

Now, $A^m = (B+I)^m$ (if A^m is required)

& thus expanding this, only 3 terms will be there and it (A^m) can easily be found out.

Ex. $A^{10} = (B+I)^{10}$

$= I + 10B + 45B^2$