

Q Let  $P$  be a non-singular matrix &  $I + P + P^2 + \dots + P^n = O$  then find  $P^{-1}$

$$I + P + P^2 + \dots + P^n = O$$

multiplying by  $(I - P)$  both side.

$$(I - P)(I + P + P^2 + P^3 + \dots + P^n) = (I - P) \cdot O$$

$$I - P^{n+1} = O$$

using formula,  $1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x}$

or simply multiplying.  $\Rightarrow (1 - x)(1 + x + x^2 + \dots + x^n) = 1 - x^{n+1}$

$$P^{n+1} = I$$

$$(P^n) P = I$$

{ If  $AB = BA = I$ , then  $A^{-1} = B$  }

Hence,  $P^{-1} = P^n$

Ans.