14. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identify matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) **TRUE**?

(A) $|FE| = |I - FE|\,|FGE|$

(B) $(I - FE)(I + FGE) = I$

(C) $EFG = GEF$

(D) $(I - FE)(I - FGE) = I$

Soln:- $G = (I-EF)^{-1}$

$\Rightarrow \quad G(I-EF) = (I-EF)G = I$ $\quad \left\{ \begin{array}{l} A = B^{-1} \\ \text{then} \\ AB = BA = I \end{array} \right\}$

$\Rightarrow \quad G - GEF = G - EFG = I$

So, $\boxed{GEF = EFG}$ → option (c) correct.

& $G - EFG = I$

$\boxed{I - G + EFG = 0}$ ⟶ (1)

Now, $(I-FE)(I+FGE) = I - FE + FGE - FEFGE$

$= I - F(E - GE + EFGE)$

$= I - F(I - G + EFG)E$

$= I - F(0)E$ {from (1)}

$= I$

option (b) is correct.

$(I-FE)(I-FGE) = I - FE - FGE + FEFGE$

$= I - F(I + G - EFG)E$

$\left\{ \begin{array}{l} \text{from (1)} \\ I - G + EFG = 0 \\ I - G = -EFG \end{array} \right\}$

$= I - F(I + G + I - G)E$

$= I - 2FE$ (d) Incorrect.

$$(I-FE)(I-FGE) = I-2FE \quad \left\{\text{Proved in previous line}\right\}$$

$$(I-FE)-(FGE)(I-FE) = I-FE-FE$$

$$-(FGE)(I-FE) = -FE$$

$$FE = (I-FE)(FGE)$$

$$|FE| = |(I-FE)(FGE)|$$

$$\boxed{|FE| = |I-FE|\,|FGE|}$$

$$\left\{ |AB| = |A|\,|B| \right\}$$

option (a) is correct.

Hence, A, B, C ⇒ correct.