Q- Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

such that $AB = B$ and $a+d = 2021$, then the value of $ad - bc$ is ___.

Soln-

$$AB = B$$

$$AB - B = 0$$

$$(A - I)B = 0 \qquad \left\{ \begin{array}{l} B \neq 0 \\ \text{hence, } |A - I| = 0 \end{array} \right\}$$

$$|A - I| = 0$$

$$\begin{vmatrix} a-1 & b \\ c & d-1 \end{vmatrix} = 0$$

$$(a-1)(d-1) - bc = 0$$

$$ad + 1 - (a + d) - bc = 0$$
$$ad - bc = (a + d) - 1$$
$$= 2021 - 1$$
$$\boxed{ad - bc = 2020}$$