Q. 35 If x, y and z are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then the value of $x^{-1} + y^{-1} + z^{-1}$ is

(a) xyz (b) $x^{-1}y^{-1}z^{-1}$ (c) $-x - y - z$ (d) -1

Sol. *(d)* We have,

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$,

$$\Rightarrow \begin{vmatrix} x & 0 & 1 \\ 0 & y & 1 \\ -z & -z & 1+z \end{vmatrix} = 0$$

Expanding along R_1,

$$x[y(1+z) + z] - 0 + 1(yz) = 0$$

$$\Rightarrow \quad x(y + yz + z) + yz = 0$$

$$\Rightarrow \quad xy + xyz + xz + yz = 0$$

$$\Rightarrow \quad \frac{xy}{xyz} + \frac{xyz}{xyz} + \frac{xz}{xyz} + \frac{yz}{xyz} = 0 \quad \text{[on dividing } (xyz) \text{ from both sides]}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} + 1 = 0$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -1$$

$$\therefore \quad x^{-1} + y^{-1} + z^{-1} = -1$$