21. If A is a symmetric matrix and B is a skew-symmetrix matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

[April 12, 2019 (I)]

(a) $\begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

(d) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

Sol:- Concept:- Any matrix 'C' can be written as the sum of a symmetric & a skew symmetric matrix.

$$C = A + B$$

where $A = \frac{C + C'}{2}$ ⇒ Symmetric matrix

$B = \frac{C - C'}{2}$ ⇒ Skew-symmetric matrix.

Now, let's assume,

$$C = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

$$A = \frac{C + C'}{2}$$

$$B = \frac{C - C'}{2}$$

$$2A = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 8 \\ 8 & -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$$

$$2B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

Hence, (b) is correct.