

27. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to : [2012]
- (a) -2 (b) 1 (c) 0 (d) -1

Soln-

$$\begin{aligned} P &\neq Q & \cancel{P^2 = Q^2} &\Rightarrow P - Q \neq 0 \\ P^3 &= Q^3 & & \\ P^2 Q &= Q^2 P & & \text{--- (1)} \\ & & & \text{--- (2)} \end{aligned}$$

By (1) - (2)

$$\begin{aligned} P^2(P - Q) &= Q^2(Q - P) \\ P^2(P - Q) &= -Q^2(P - Q) \\ P^2(P - Q) + Q^2(P - Q) &= 0 \\ (P^2 + Q^2)(P - Q) &= 0 \end{aligned}$$

$$P - Q \neq 0$$

$$\text{So, } |P^2 + Q^2| = 0$$

Concept used-

$$\left\{ \begin{array}{l} \text{If } AB = 0 \\ \& B \neq 0 \text{ then } |A| = 0 \end{array} \right\}$$