Example 6 Prove that $(A^{-1})' = (A')^{-1}$, where A is an invertible matrix.

Solution Since A is an invertible matrix, so it is non-singular.

We know that $|A| = |A'|$. But $|A| \neq 0$. So $|A'| \neq 0$ i.e. A' is invertible matrix.

Now we know that $AA^{-1} = A^{-1} A = I$.

Taking transpose on both sides, we get $(A^{-1})'$ $A' = A' (A^{-1})' = (I)' = I$

Hence $(A^{-1})'$ is inverse of A', i.e., $(A')^{-1} = (A^{-1})'$