Q. 52 Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Thinking Process

We know that, any square matrix A can be expressed as the sum of a symmetric matrix and skew-symmetric matrix, i.e., $A = \frac{A + A'}{2} + \frac{A - A'}{2}$, *where* $A + A'$ *and* $A - A'$ *are a symmetric matrix and a skew-symmetric matrix, respectively.*

Sol. We have,

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$$

$\therefore$

$$A' = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now,

$$\frac{A + A'}{2} = \frac{1}{2}\begin{bmatrix} 4 & 4 & 5 \\ 4 & -2 & 3 \\ 5 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix}$$

and

$$\frac{A - A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

$\therefore$

$$\frac{A + A'}{2} + \frac{A - A'}{2} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

which is the required expression.