

Q. 64 If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

- (a) A (b) $I - A$ (c) $I + A$ (d) $3A$

Sol. (a) We have, $A^2 = I$

$$\begin{aligned} \therefore (A - I)^3 + (A + I)^3 - 7A &= [(A - I) + (A + I)\{(A - I)^2 \\ &\quad + (A + I)^2 - (A - I)(A + I)\}] - 7A \\ &\qquad\qquad\qquad [\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)] \\ &= [(2A)\{A^2 + I^2 - 2AI + A^2 + I^2 + AI - (A^2 - I^2)\}] - 7A \\ &= 2A[I + I^2 + I + I^2 - A^2 + I^2] - 7A \qquad\qquad\qquad [\because A^2 = AI] \\ &= 2A[5I - I] - 7A \\ &= 8AI - 7AI \\ &= AI = A \qquad\qquad\qquad [\because A = AI] \end{aligned}$$