

Q6: A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to 1/1000 of the original amplitude is close to

- (a) 100 s
- (b) 10 s
- (c) 20 s
- (d) 50 s

SOLUTION :

from the data, we have

$$f = 5 \text{ Hz}$$

so, for Amplitude to decrease to half
Time taken = 2 sec of as 10 oscillations
will take 2 sec

so, again using damping amplitude
formula.

$$A = A_0 e^{-\lambda t} \quad \text{for } \lambda = \frac{bt}{2m}$$

$$\text{now at } t = t_1 = 2 \text{ sec, } A = \frac{A_0}{2}$$

$$\text{so, } \frac{A_0}{2} = A_0 e^{-\lambda t} \Rightarrow \ln 2 = \lambda t \Rightarrow \boxed{\lambda = \frac{\ln 2}{2}}$$

$$\text{for } A = \frac{A_0}{1000} \text{ we have } \frac{A_0}{1000} = A_0 e^{-\lambda t} \Rightarrow \frac{\ln(1000)}{\lambda} = t$$

$$\text{so, } t = \frac{2 \times \ln(1000)}{\ln(2)} = \frac{2 \times 3 \times \ln(10)}{\ln(2)} \approx \boxed{20 \text{ sec}}$$