JEE MAINS – 2021 20th July Evening

A particle is making simple harmonic motion along the X-axis. If at distances x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as:

(1) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$

(2) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$

(3) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$

(4) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

SOLUTION :

we know that

$$v = \omega\sqrt{A^2 - x^2}$$

so, $v_1 = \omega\sqrt{A^2 - x_1^2}$ $\qquad$ $v_2 = \omega\sqrt{A^2 - x_2^2}$

so, $\dfrac{v_1^2}{v_2^2} = \dfrac{A^2 - x_1^2}{A^2 - x_2^2}$

$$\frac{v_1^2}{v_2^2} - 1 = \frac{A^2 - x_1^2}{A^2 - x_2^2} - 1$$

$$\frac{v_1^2 - v_2^2}{v_2^2} = \frac{A^2 - x_1^2 - A^2 + x_2^2}{A^2 - x_2^2}$$

$$\frac{v_1^2 - v_2^2}{v_2^2} = \frac{x_2^2 - x_1^2}{A^2 - x_2^2}$$

which gives

$$\frac{v_2^2}{A^2 - x_2^2} = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}$$

now $v_2 = \omega\sqrt{A^2 - x_2^2} \Rightarrow \omega = \dfrac{v_2}{\sqrt{A^2 - x_2^2}}$

which gives

$$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2} \Rightarrow \omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}}$$

NOW $\quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$ $\quad$ WHICH IS OPTION(4)