A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in the same direction, then:

(1) Amplitude of motion is 4a

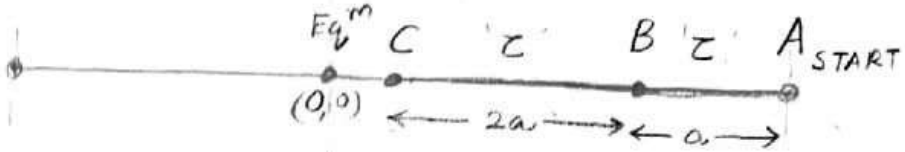
(2) Time Period of oscillation is 6τ

(3) Amplitude of motion is 3a

(4) Time period of oscillation is 8τ

SOLUTION :

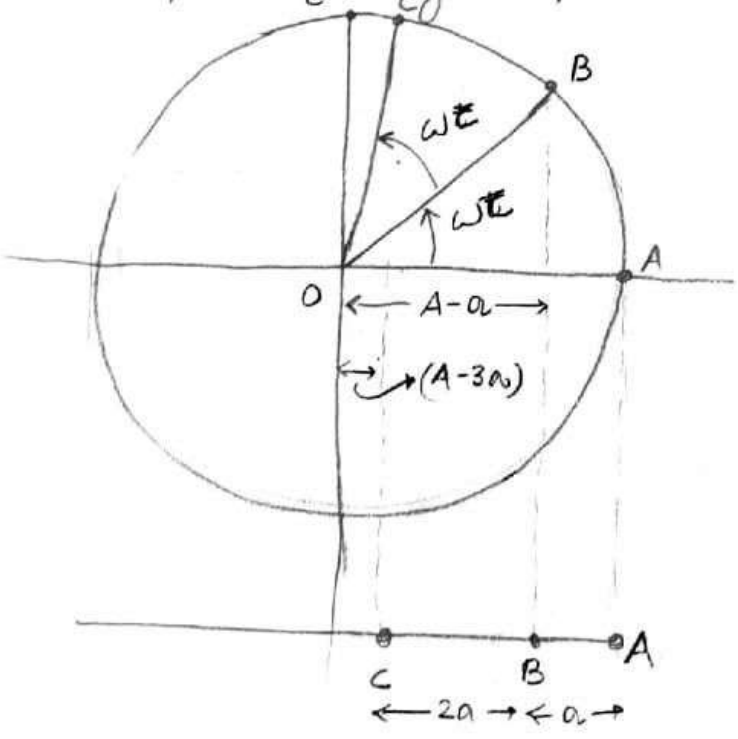
first point to note, the particle starts from rest, which implies that particle starts from extreme position.



Since, particle starts from extreme position.

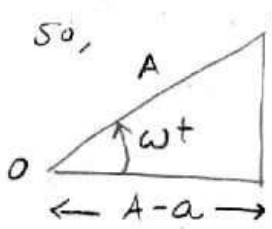
So, equation is $x = A\cos\omega t$

Let's use phasor diagram,

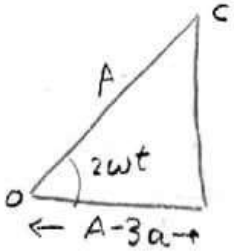


Since time from 'A' to B is 'τ' & also for 'B' to 'C'

So,



$\cos\omega t = \frac{A-a}{A}$



$\cos 2\omega t = \frac{A-3a}{A}$

So, from eq^n (I) & (II)

$$\cos\omega t = \frac{A-a}{A} \quad \& \quad \cos 2\omega t = \frac{A-3a}{A}$$

$$\cos 2\omega t = 2\cos^2\omega t - 1 \Rightarrow \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\frac{A-3a}{A} = \frac{2A^2+2a^2-4Aa-A^2}{A^2}$$

$$A^2 - 3Aa = A^2 + 2a^2 - 4Aa$$

$$2a^2 = Aa \Rightarrow \boxed{A = 2a}$$

which gives

$$\frac{A-a}{A} = \cos\omega\tau \Rightarrow \frac{a}{2a} = \cos\omega\tau$$

$$\cos\omega\tau = \frac{1}{2} \Rightarrow \omega\tau = \frac{\pi}{3} \Rightarrow \tau = \frac{\pi}{3\omega}$$

and $T = \frac{2\pi}{\omega} \Rightarrow \frac{2\pi}{\pi} \times 3\tau = 6\tau$

which gives $\boxed{T = 6\tau}$

correct answer (b)