

Q) If $a, b, c \in \mathbb{R}$, $a \neq 0$ & $(b-1)^2 < 4ac$, then the number of real roots of the system of equations

[in three unknowns x_1, x_2, x_3]

$$ax_1^2 + bx_1 + c = x_2, \quad ax_2^2 + bx_2 + c = x_3, \quad ax_3^2 + bx_3 + c = x_1$$

- (a) 0 (b) 1 (c) 2 (d) 3

Soln Given equation $(b-1)^2 < 4ac$

let $f(x) = ax^2 + (b-1)x + c$

\therefore The given 3 equations are written as

$$f(x_1) = x_2 - a_1 \quad \text{--- (1)}$$

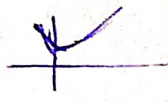
$$f(x_2) = x_3 - a_2 \quad \text{--- (2)}$$

$$f(x_3) = x_1 - a_3 \quad \text{--- (3)}$$

Now from (1) + (2) + (3) = 0

$$\therefore f(x_1) + f(x_2) + f(x_3) = 0$$

$$\Rightarrow a f(x_1) + a f(x_2) + a f(x_3) = 0$$

given $(b-1)^2 < 4ac \Rightarrow$ 

graph won't touch the x-axis

$$\rightarrow a f(x_1) > 0, \quad a f(x_2) > 0, \quad a f(x_3) > 0$$

So, the given system of equations has

no real roots.

option 'A' is correct.