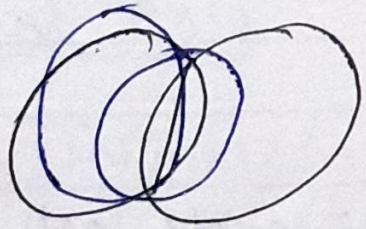
Family of Circles :

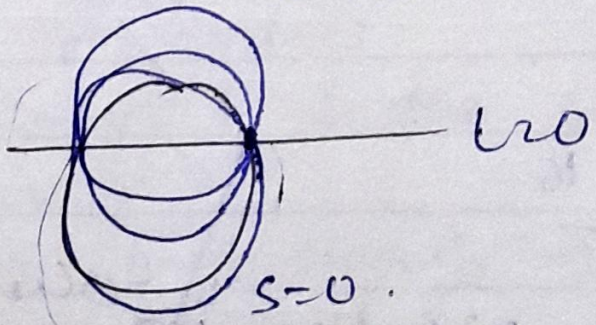
① Eqn of family of circle passing through pt of intersection of two circle $S_1 = 0$ & $S_2 = 0$ is given by

$S_1 + \lambda S_2 = 0$ where ~~belong to~~ $\lambda \in R - \{-1\}$.



② Eqn of family of circle passing through pts of intersection of circle $S = 0$ & line $L = 0$ is given by

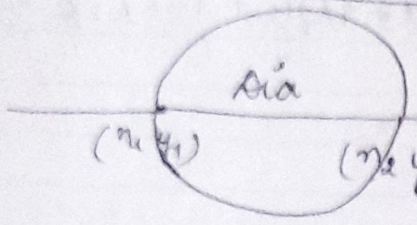
$\boxed{S + \lambda L = 0}$ where $\lambda \in R$.



(3) Eqn of family of circle passing through a fixed pt. $A(x_1, y_1)$ & $B(x_2, y_2)$ is given by

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+\lambda\left[y-y_1-\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)\right]=0 \quad \lambda \in R$$

$$S + \lambda L = 0$$



$S=0$.

$$y-y_1-\frac{y_2-y_1}{x_2-x_1}(x-x_1)\geq 0.$$

$L \geq 0$

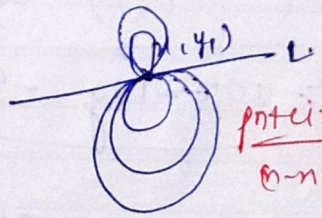
(4) Eqn of family of circle touching a fixed lines $y-y_1 = m(x-x_1)$ at a fixed point $A(x_1, y_1)$. is given by $(x-x_1)^2+(y-y_1)^2+\lambda[(y-y_1)-m(x-x_1)] \geq 0$.

S $\qquad$ L

$S + \lambda L \geq 0$

↓

pointcircle line.



pntcirele

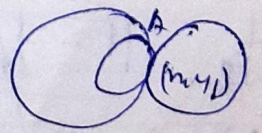
$(x-x_1)^2+(y-y_1)^2 \geq 0 \Rightarrow \sqrt{S \geq 0}$

(5) eqn of family of circle touching a fixed circle $S_1 \geq 0$ at a fixed point A is given by. $A(x_1, y_1)$.

$$S_1 + \lambda\left[(x-x_1)^2+(y-y_1)^2\right] \geq 0 \quad \lambda \in R-\{-1\}$$

$S_1 + \lambda S_2 \geq 0$

Point circle.



Ques. find eqn of circle passing through pnt of int. of circle $x^2+y^2+6x+2y+4=0$ & $x^2+y^2+2x-4y-6=0$.
centre lies on line $y=x$.

$$x^2+y^2-6x+2y+4+\lambda(x^2+y^2+2x-4y-6)=0.$$

$$(1+\lambda)x^2+(1+\lambda)y^2+(-6+2\lambda)x+(2-4\lambda)y+4-6\lambda=0.$$

centre $y=x$.

$$\text{centre} = \left(\frac{-6+2\lambda}{2}, \frac{(2-4\lambda)}{2}\right)$$

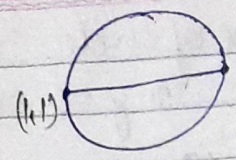
$$-3+\lambda = 1-2\lambda.$$

$$3\lambda = 4$$

$$\boxed{\lambda = \frac{4}{3}}$$

$$\frac{7}{3}x^2+\frac{7}{3}y^2+\left(-6+2\left(\frac{4}{3}\right)\right)x+\left(2-\frac{16}{3}\right)y+4-6\left(\frac{4}{3}\right)=0.$$

$$\boxed{7x^2+7y^2-10x-10y-4=0.}$$



$S=0$ $\quad (x-1)(x-2)+(y-1)(y-2)=0.$

$x^2+y^2-3x-3y+4=0.$

eqn of family of circles is $S+\lambda L=0.$

$x^2+y^2-3x-3y+4+\lambda(x-y)=0.$

$x^2+y^2+(\lambda-3)x-(\lambda+3)y+4=0.$

$g=\frac{\lambda-3}{2}$ $\quad f=-\left(\frac{\lambda+3}{2}\right)$ $\quad c=4.$

$R=1$

$g^2+f^2-c=1$

$\left(\frac{\lambda-3}{2}\right)^2+\left(\frac{\lambda+3}{2}\right)^2-4=0$

$\boxed{\lambda=\pm 1}$

find eqn of circle pass through (2,1) & touching the line $x+2y-1=0.$ at pnt $(3,-1).$

point circle ~~$(x-2)^2+(y-1)^2=0$~~

~~$(x-2)^2+(y-1)$~~ $(x-3)^2+(y+1)^2+\lambda(x+2y-1)=0.$

$1+4+\lambda(2+2-1)=0$

$5+\lambda(3)=0$

$\boxed{\lambda=-\frac{5}{3}}$

$\boxed{(x-3)^2+(y+1)^2-\left(\frac{5}{3}\right)(x+2y-1)=0.}$

Q. find eqⁿ of circle which bisects the circumference of circle $x^2+y^2+2y-3=0$ & touches the line $x-y=0$ at $(0,0)$

eqⁿ of family of circle

$$x^2+y^2+\lambda(x-y)=0$$

point circle

$$x^2+y^2=0.$$

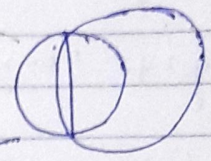
common chord eqⁿ

$$x^2+y^2+2y-3-x^2-y^2-\lambda(x-y)=0$$

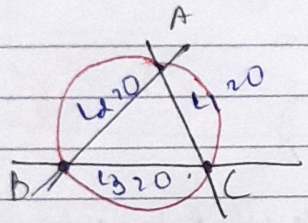
$$-\lambda x+(2+\lambda)y-3=0.$$

$C(0,-1)$

$$\lambda=-5$$



NOTE.



eqⁿ of circle circumscribing the Δ formed by the three lines.

$L_1=0$, $L_2=0$, $L_3=0$ is given by.

(3 non ll and non concurrent lines).

$$L_1L_2+\lambda L_2L_3+\mu L_3L_1=0$$

provided.

coeffⁿ of x^2 = coeff of y^2

coeff of $xy=0$.

(2) eqn of circle circumscribing a quads formed by three lines $L_1=0$ $L_2=0$ $L_3=0$, $L_4=0$ (taken in order only) is given by $\lambda L_1 L_3 + \mu L_3 L_4 = 0$

provided $x^2(\text{coeff}) = \text{coeff } y^2$
coeff of $xy = 0$