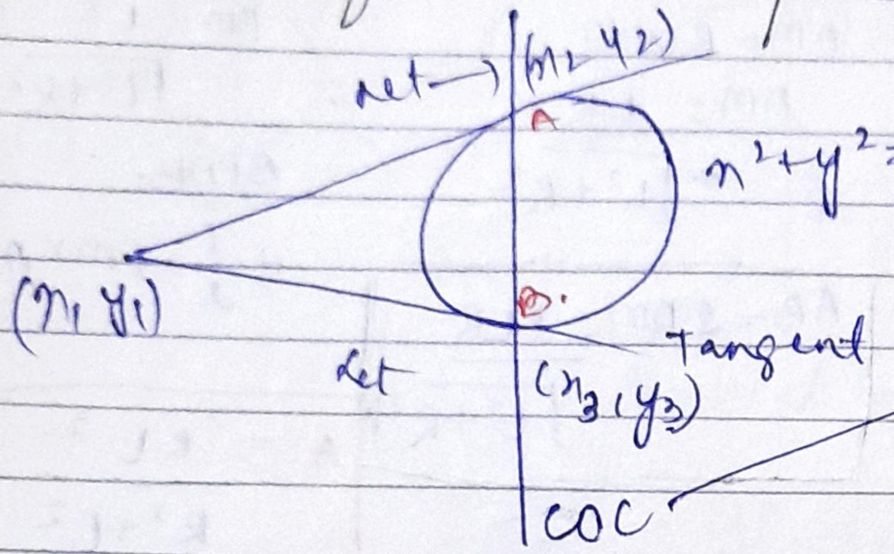
Chord of Contact / (COC) :-



eqn of COC is $\boxed{T=0}$

eqn of Tangent at A (x_2, y_2) is $T=0$.

$$x x_2 + y y_2 = r^2$$

It passes through (x_1, y_1)

$$x_1 x_2 + y_1 y_2 = r^2$$

$$x_1 x_3 + y_1 y_3 = r^2$$

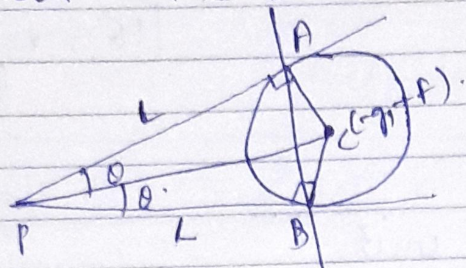
$\Rightarrow x_1 x + y_1 y = r^2$ → $(x_2 y_2)$, $(x_3 y_3)$

$\parallel$ COC

∴ eqn can be written as $T=0$

Some Important :

Chord of contact exist only when pnt does not lies inside the circle.



$x^2+y^2+2gx+2fy+c=0$.

$L = \sqrt{S_1}$

$PC = \sqrt{L^2+R^2}$

$\cos\theta = \frac{L}{\sqrt{L^2+R^2}}$

$\sin\theta = \frac{R}{\sqrt{L^2+R^2}}$

~~cos~~ $\tan\theta = \frac{R}{L}$

ΔACM :-

$\cos\theta = \frac{AM}{R}$

$AM = R\cos\theta$.

$AM = \frac{LR}{\sqrt{L^2+R^2}}$

$\boxed{AB = 2AM = \frac{2LR}{\sqrt{L^2+R^2}}}$

ΔPAM :-

$\cos\theta = \frac{PM}{L}$

$PM = \frac{L^2}{\sqrt{L^2+R^2}}$

ΔPAB =

$\frac{1}{2} \times PM \times AB$

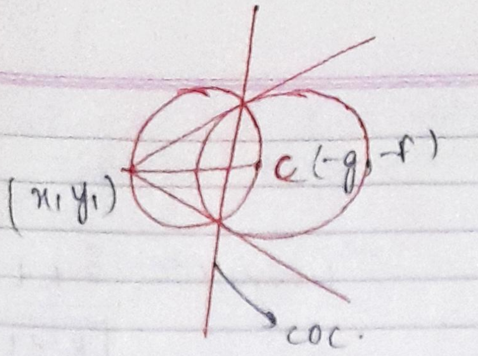
$\boxed{A = \frac{RL^3}{R^2+L^2}}$

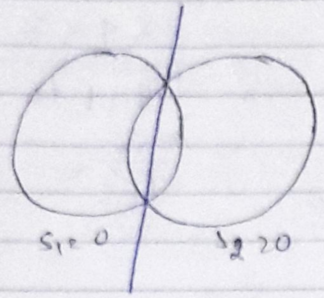
(2) Length of COC $= AB = \frac{2RL}{\sqrt{L^2+R^2}}$.

(3) Area of Δ formed by pair of tangents and COC $= \frac{RL^3}{L^2+R^2}$

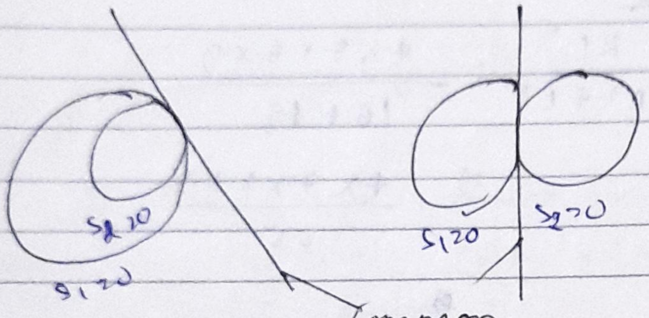
(4) Eq^n of circle circumscribing the Δ formed by pair of tangents drawn from pt $P(x_1, y_1)$ and COC is

$$\boxed{(x-x_1)(x+g)+(y-y_1)(y+f)=0}$$





eqn of common chord. $\boxed{S_1 - S_2 = 0}$ when coeff of x^2 & $y^2 = 1$ in both circle.



Common Tangent at point of contact of two circle

eqn is $\boxed{S_1 - S_2 = 0}$

Ques. Consider the eqn of circle. $R = \sqrt{4+9+3}$

$x^2 + y^2 + 6x - 4y - 3 = 0$, from the pnt $(5, 1)$ Tangents are drawn to the circle whose point of contact are A & B. find

(i) eqn of COC.

(ii) Length of COC

(iii) Area of Δ formed by pair of tangent & their COC.

(iv) Eqn of circle circumscribing Δ PAB.

eqn of COC,

$$5x + y + 3(x+5) - 4(y+1) - 3 = 0$$

$$5x + y + 3x + 15 - 4y - 4 - 3 = 0.$$

$8x - 3y + 15 - 7 = 0$

$8x - 3y + 8 = 0.$

$L \Rightarrow \quad 25 + 1 + 30 - 4 - 3.$

$26 + 30 - 7.$

$56 - 7.$

$\sqrt{49} = 7$ unit

length COC $= \dfrac{2RL}{\sqrt{R^2 + L^2}}$

$= \dfrac{2 \times 4 \times 7}{\sqrt{49 + 16}}$

$\Rightarrow \dfrac{56}{\sqrt{65}}$

~~Area of Δ = $\frac{1}{2} \times R \times L$~~

~~$= \frac{1}{2} \times 7 \times 4 = 14$ sq. unit~~

~~eqn of circle circum~~

$\dfrac{RL^3}{R^2 + L^2} = \dfrac{4 \times 7 \times 7 \times 7}{16 + 49}$

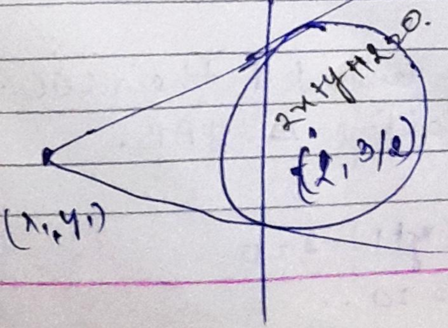
$\Rightarrow \dfrac{4 \times 7 \times 7 \times 7}{65}$

Important

~~Area of~~ eqn of circle is

$$\boxed{(x-5)(x+3) + (y-1)(y-2) = 0.}$$

Find the coordinates of pnt of intersection of tangent at pnts where the line $2x + y + 12 = 0$ meets the circle $x^2 + y^2 - 4x + 3y - 1 = 0$.



$xx_1 + yy_1 - 2(x + x_1) + \frac{3}{2}(y + y_1) - 1 = 0.$

$xx_1 + yy_1 - 2x - 2x_1 + \frac{3}{2}y + \frac{3}{2}y_1 - 1 = 0$

$(x_1 - 2)x + \left(y_1 + \frac{3}{2}\right)y + \left(\frac{3y_1}{2} - 2x_1 - 1\right) = 0$

since eqn 1 & two represent same line therefore compare them.

$$\frac{x_1-2}{2} = \frac{y_1+3/2}{1} = \frac{-2x_1+3/2\,y_1-1}{12}$$

$x_1-2 = 2y_1+3$

$\boxed{x_1-2y_1=5}$ $\times 16$

~~$+10x_1=12$~~

$\boxed{16x_1-3y_1=22}$

$16x_1-32y_1=80$
$16x_1-3y_1=22$

$-29y_1=58$

$\boxed{y=-2}$

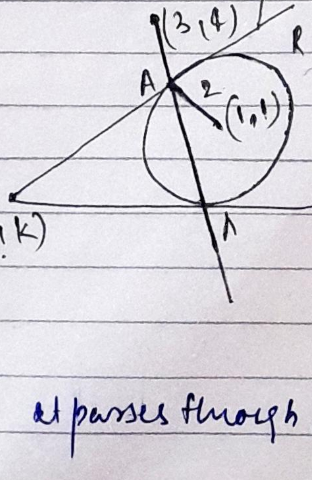
$x_1+4=5$

$\boxed{x_1=1}$

$(1,-2)$

Ques.

If a variable line is drawn from the pnt $(3,4)$ Intersecting the circle $x^2+y^2-2x-2y-2=0$. at two pnt then find locus of pnt of Intersection of Tangents at these two pnts.



$R=\sqrt{1+1+2}$
$=2$ unit

eqn of chord of contact.

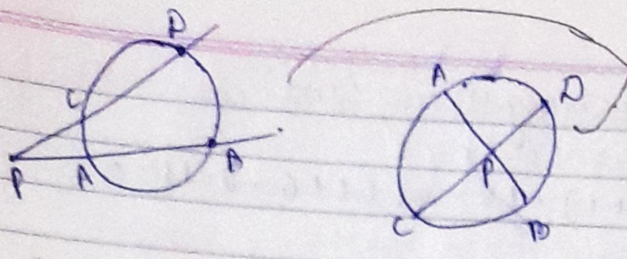
$y-4=m(x-3)$
$y-4=mx-3m$
$mx-y-3m+4=0$. (h,k)

$T=0$

$$hx+ky-2\left(\frac{x+h}{2}\right)-2\left(\frac{y+k}{2}\right)-2=0$$

it passes through $(3,4)$

$\therefore \boxed{2x+3y-9=0}$

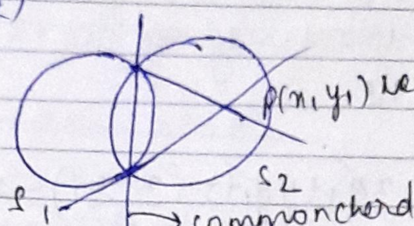


$\to PA \cdot PB = PC \cdot PD$.

Ques Tangents are drawn to circle $x^2+y^2=12$ at the pnts where it ~~A~~ meet/intersect the circle $x^2+y^2-5x+3y-2=0$. find pnt of intersection of these tangents. centre $\left(\frac{5}{2}, \frac{-3}{2}\right)$

$12-5x+3y-2=0$

~~5x+1~~ $5x-3y=10$



$S_1 - S_2 = 0$

$5x-3y-10=0 \to (2)$

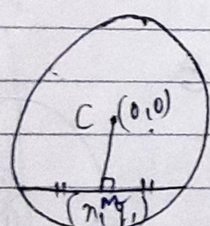
eqn of coc for s i $T=0$

$xx_1+yy_1-12=0 \to (1)$

compare eq (1) & (2)

$\frac{x_1}{5} = \frac{y_1}{-3} = \frac{-12}{-10} = \frac{6}{5}$ $\quad P\left(6, \frac{-18}{5}\right)$.

Chord with given middle point.



$m_{CM} = \frac{y_1}{x_1}$

eqn of chord →

$y-y_1 = -\frac{x_1}{y_1}(x-x_1)$

$yy_1-y_1^2 = -xx_1+x_1^2$

$xx_1+yy_1-r^2 = x_1^2+y_1^2-r^2$.

$\boxed{T = S_1}$