• TIPS & TRICKS

1> FOR FINDING TIME PERIOD OF ANY SHM PROBLEM:

STEP 1 : ANALYSE EQUILIBRIUM POINT TO GET SOME BASIC INFORMATION

STEP 2 : DISPLACE THE MASS (UNDER OBSERVATION) BY 'x' & THEN ASSIGN IT SOME ACCELERATION AND THEN DRAW FBD OF THE MASS, TO WRITE DOWN THE FORCE EQUATION, REDUCE THE EQUATION TO –

$a = -\lambda x$, SO WE KNOW THAT

$\omega^2 = \lambda$ AND $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\lambda}}$

2> FOR WRITING SHM EQUATION –

- IF PARTICLE STARTS FROM EQUILIBRIUM –

$x = \pm A\sin\omega t$

- IF PARTICLE STARTS FROM EXTREME POSITION –

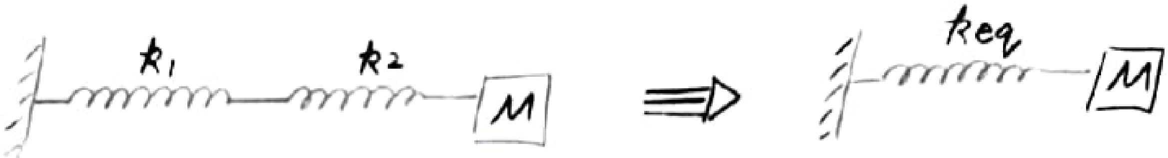
$x = \pm A\cos\omega t$

$\pm \rightarrow$ depends on initial direction of motion (velocity).

- TIPS & TRICKS

1> Combination of Springs

- **Series Combination**



- Force on both spring is same
- Extension in spring is different

$$\frac{x_1}{x_2} = \frac{k_2}{k_1} \left\{ \text{as } kx = \text{Same} \right\}$$

where $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$ or

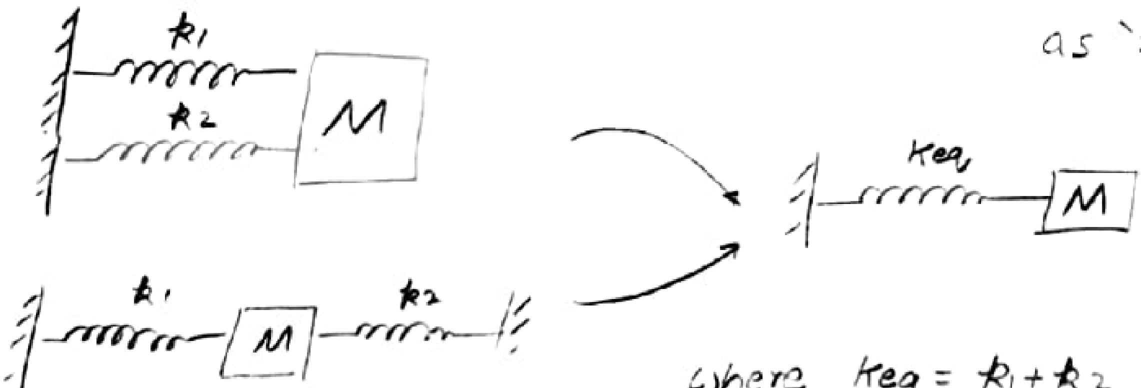
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

so, $T = 2\pi\sqrt{\dfrac{M}{k_{eq}}}$

- **Parallel combination**
  - force on both spring different
  - Extension in spring is same.

$$\frac{F_1}{F_2} = \frac{k_1}{k_2} = \frac{PE_1}{PE_2}$$

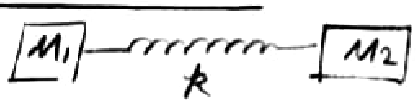
as 'x' is same



where $k_{eq} = k_1 + k_2$

so, $T = 2\pi\sqrt{\dfrac{M}{k_{eq}}}$

- **When two masses**



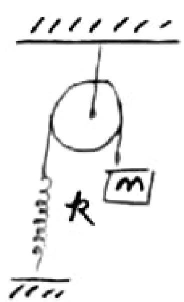
use $M = \dfrac{M_1 M_2}{M_1 + M_2}$ { REDUCED MASS }

so, $T = 2\pi\sqrt{\dfrac{M}{k}}$

SOME SPRING MASS SYSTEM :

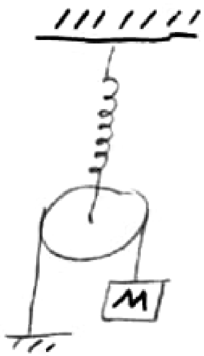
{FOR SOLVING THESE USE 2-STEP ANALYSIS MENTIONED IN TIPS & TRICKS PART OF LECTURE-1}

1>



$$T = 2\pi\sqrt{\frac{m}{k}}$$

11>



$$T = 2\pi\sqrt{\frac{4M}{k}}$$

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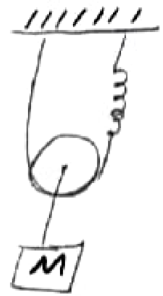
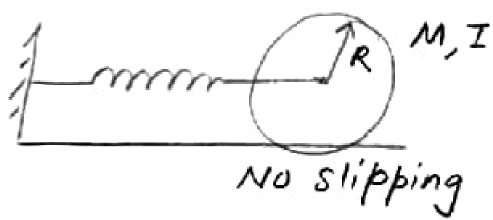


$$T = 2\pi\sqrt{\frac{m}{4k}}$$

1V>



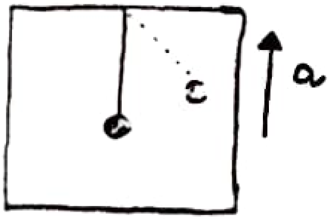
$$T = 2\pi\sqrt{\frac{M + I/R^2}{k}}$$

$I \equiv$ Moment of Inertia

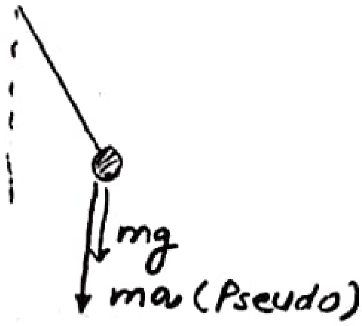
TIPS & TRICKS

- SIMPLE PENDULUM IN LIFT {ACCELERATING PENDULUM IN LIFT}:

1>



If we look at the fbd of mass, we will need to find the total force on mass {including pseudo force}



$F = mg + ma$

$mg_{eff} = mg + ma$

$g_{eff} = g + a$

2nd way:

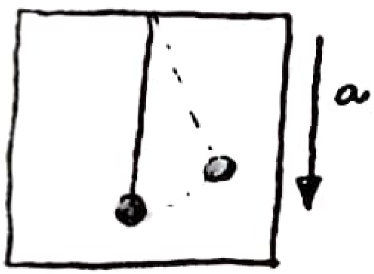
Take $g_{eff} = |\vec{g} - \vec{a}|$ Now if ↓ is taken negative

so, $\vec{g} = -g$ & $\vec{a}_p = +a$

so, $g_{eff} = |-g-a| = g+a$

so, $T = 2\pi\sqrt{\dfrac{l}{g_{eff}}} = 2\pi\sqrt{\dfrac{l}{g+a}}$

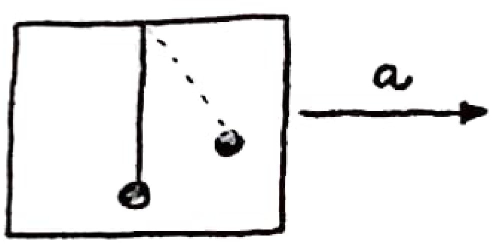
2>



$T = 2\pi\sqrt{\dfrac{l}{g-a}}$

In case of free fall $a=g$, $T=\infty$

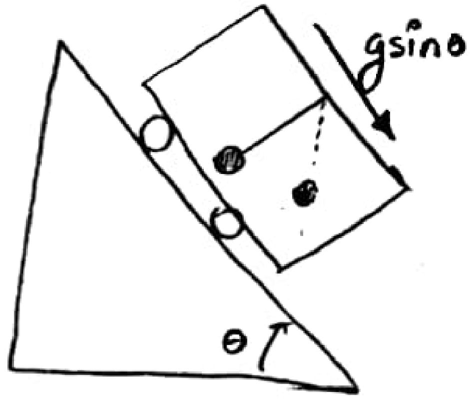
3>



$g_{eff} = \sqrt{g^2+a^2}$

so, $T = 2\pi\sqrt{\dfrac{l}{\sqrt{g^2+a^2}}}$

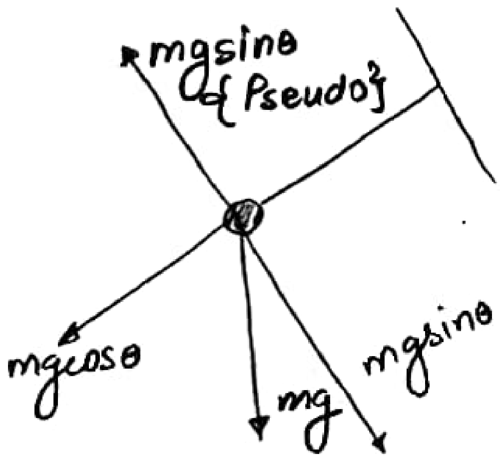
<4>



$$g_{eff} = g\cos\theta$$

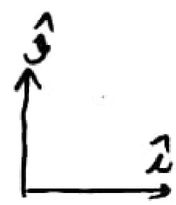
$$T = 2\pi\sqrt{\frac{l}{g\cos\theta}}$$ { SOMEWHAT TRICKY }

<u>TAKING FBD OF MASS</u> { INCLUDING PSEUDO FORCE due to $g\sin\theta$ }



Total $g_{eff} = g\cos\theta$

2^{nd} way

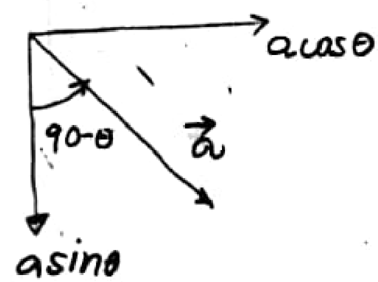


$$g_{eff} = |\vec{g} - \vec{a}|$$

here $\vec{g} = -g\hat{j}$,

NOW $\vec{a}$

$= a\cos\theta\,\hat{i}$
$- a\sin\theta\,\hat{j}$



so, NOW

$$g_{eff} = |-g\hat{j} - a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}|$$

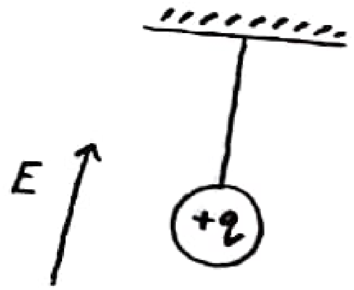
$$= \sqrt{a^2\cos^2\theta + a^2\sin^2\theta + g^2 - 2ag\sin\theta}$$

$$= \sqrt{a^2 + g^2 - 2ag\sin\theta} \quad \text{as } \boxed{a = g\sin\theta}$$

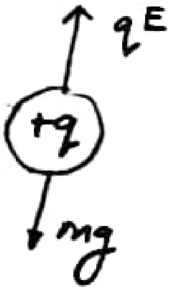
$$= \sqrt{g^2\sin^2\theta + g^2 - 2g^2\sin^2\theta}$$

$$= \sqrt{g^2 - g^2\sin^2\theta} = g\cos\theta$$

<5>



so,

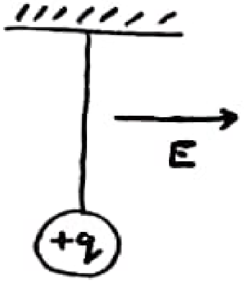


$F = mg - qE$

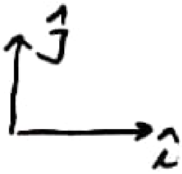
$g_{eff} = g - \frac{qE}{m}$

$$T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$$

<6>



Using $g_{eff} = |\vec{g} - \vec{a}|$



$\vec{g} = g(-\hat{j})$ $\quad \vec{a} = \frac{qE}{m}\hat{i}$

so, $g_{eff} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$

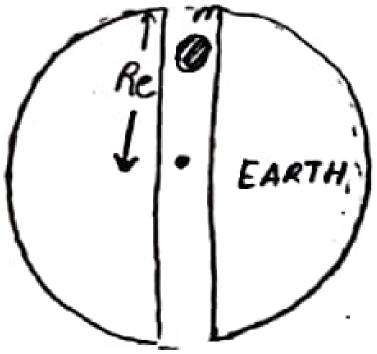
so, $$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{q^2E^2}{m^2}}}}$$

* SOME SPECIAL CASES

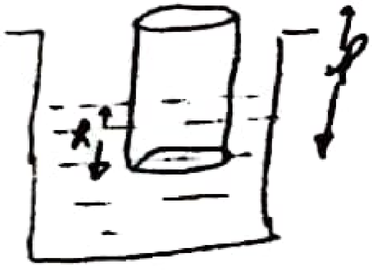
* Tunnelling the Earth

$$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6 \text{ min}$$

Amplitude $\leq R_e$



* Oscillation of Floating Cylinder

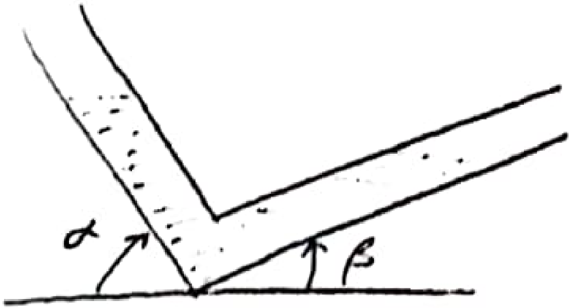


$\sigma \equiv$ density of cylinder

$\rho \equiv$ density of fluid $(\sigma > \rho)$

$$T = 2\pi\sqrt{\frac{\sigma L}{\rho g}} = 2\pi\sqrt{\frac{h}{g}}$$

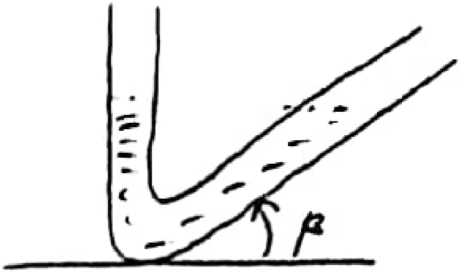
* Oscillation of Liquid In a Tube



$$T = 2\pi\sqrt{\frac{l}{g(\sin\alpha + \sin\beta)}}$$

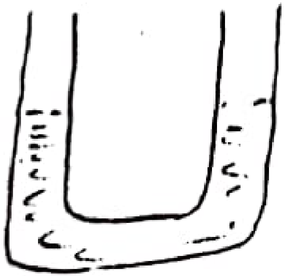
l = Total length of liquid column.

1> when $\alpha = 90^\circ$



$$T = 2\pi\sqrt{\frac{l}{g\sin\beta + g}}$$

2> when $\alpha = \beta = 90^\circ$



$$T = 2\pi\sqrt{\frac{l}{2g}}$$