1. PERIODIC MOTION

When a body or a moving particle repeats its motion along a definite path after regular intervals of time, its motion is said to be **Periodic Motion**.

2. OSCILLATORY MOTION

If a particle moves back and forth (to and fro) over the same path periodically then its motion is said to be **oscillatory or vibratory** e.g., motion of a pendulum.

Note : Every oscillatory motion is periodic but every periodic motion is not oscillatory. For example, motion of earth around the sun is periodic but not oscillatory, and the motion of pendulum is oscillatory as well as periodic.

3. SIMPLE HARMONIC MOTION

If the restoring force/torque acting on the body in oscillatory motion is directly proportional to the displacement of body/particle and is always directed towards equilibrium position then the motion is called Simple Harmonic Motion (SHM).

Linear SHM – When a particle moves to and fro about an equilibrium point, along a straight line.

Angular SHM – When body/particle is free to rotate about a given axis executing angular oscillations.

4. EQUATION OF SIMPLE HARMONIC MOTION (SHM)

The necessary and sufficient condition for SHM is $\boldsymbol{F} = -\boldsymbol{kx}$ where k = positive Force constant

SOLUTION : $x = Asin(\omega t + \varphi)$ where φ is the initial phase.

Example – When the particle starts from extreme position and not equilibrium position, we will have $x = A\ at\ t = 0$ so which will give $\varphi = \pm 90°$ so the equation becomes $x = \pm Acos(\omega t)$

5. CHARACTERISTICS OF SHM

(a) **Amplitude (A)** – Maximum value of displacement of the particle from its equilibrium position. It depends on energy of the system.

(b) **Time Period (T)** – Smallest time interval after which the oscillatory motion gets repeated.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

(c) **Phase Constant ($\boldsymbol{\varphi}$)** – Depends on the initial position and direction of velocity.

DISPLACEMENT, VELOCITY AND ACCELERATION IN SHM

Displacement $x = Asin(\omega t + \varphi)$

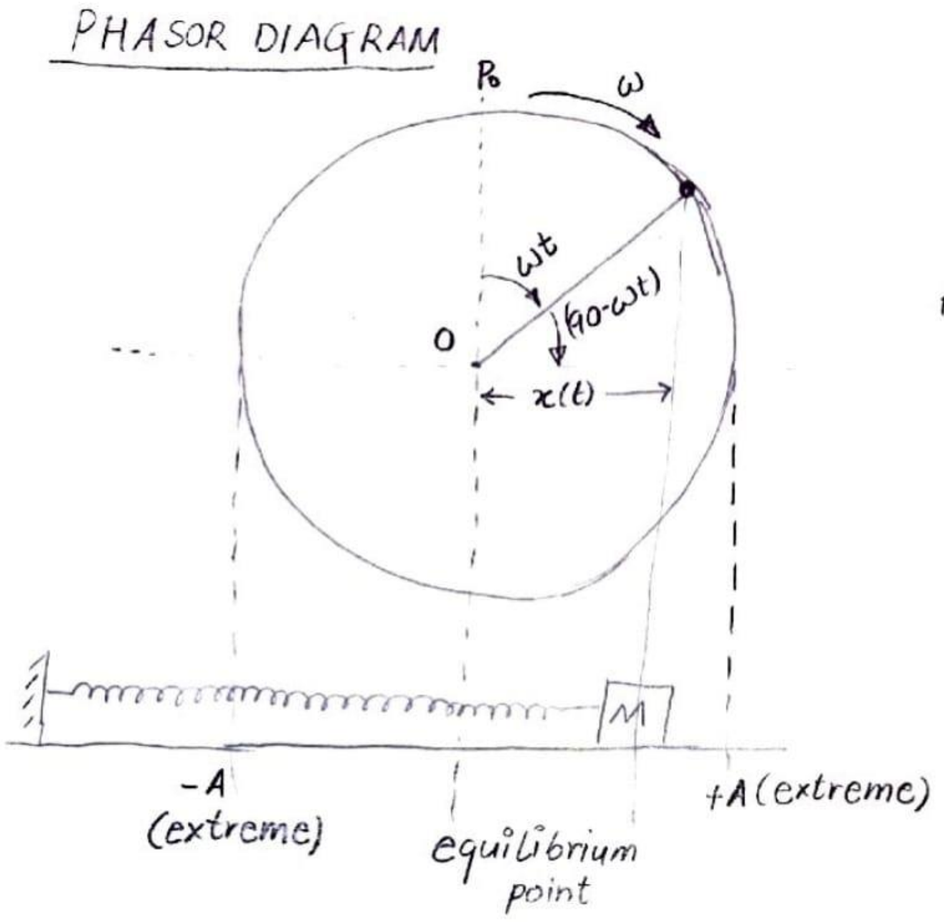
Velocity $v = A\omega cos(\omega t + \varphi) = A\omega sin\left(\omega t + \varphi + \frac{\pi}{2}\right)$ or finally $v = \omega\sqrt{A^2 - x^2}$

Acceleration $x = -\omega^2 Asin(\omega t + \varphi) = \omega^2 Asin(\omega t + \varphi + \pi)$ or finally $a = -\omega^2 x$

Time , t	0 (Mean Position)	T/4 (Extreme Position)	T/2 (Mean Position)	3T/4 (Extreme Position)	T (Mean Position)
Displacement, x	0	A	0	-A	0
Velocity, v	$A\omega$	0	$-A\omega$	0	$A\omega$
Acceleration, a	0	$-\omega^2 A$	0	$\omega^2 A$	0

Graph of Velocity vs Displacement will be Elliptical.

SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION



from Triangle

$\cos(90 - \omega t) = \frac{x(t)}{A}$

$x(t) = A\sin(\omega t)$

ENERGY OF SHM

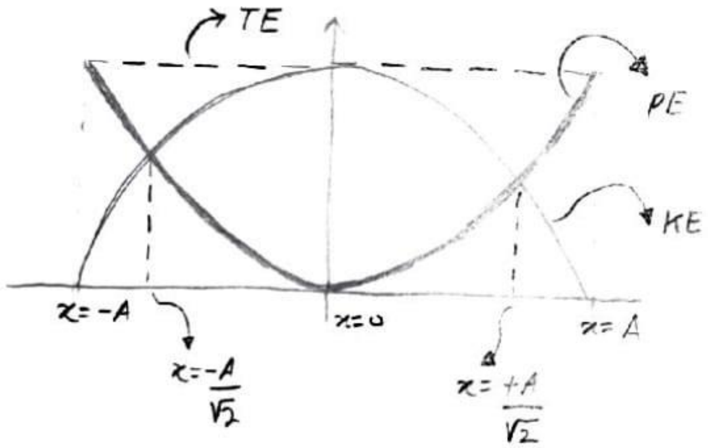
- KINETIC ENERGY (KE)
  - $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2-x^2) = \frac{1}{2}k(A^2-x^2)$ as $\omega^2 = \frac{k}{m}$
  - $\frac{1}{2}mv^2 = \frac{1}{2}kA^2\cos^2(\omega t+\phi)$ AS $v = A\omega\cos(\omega t+\phi)$
  - $KE_{max} = \frac{1}{2}kA^2$ when v is max – at mean position.

* frequency of KE = 2 (frequency of SHM)
  i.e Graph of KE completes two cycle in one Time Period of SHM.

- POTENTIAL ENERGY (PE)
  $\frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t+\phi)$ $PE_{max} = \frac{1}{2}kA^2$ at extreme position.
  - Same frequency as KE.

- TOTAL MECHANICAL ENERGY (ME)
  $ME = KE + PE$
  $= \frac{1}{2}k(A^2-x^2) + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ constant throughout the motion.

GRAPH



The Potential Energy & Kinetic Energy are equal to $x = \pm\frac{A}{\sqrt{2}}$

SIMPLE PENDULUM

If a heavy point-mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum.

Time Period of oscillation of simple pendulum of length l for small angular amplitude is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Note – We can take g = π^2 for making calculation simpler.

SPECIAL POINTS

1. **Seconds Pendulum** – T = 2s = Time Period of Seconds Pendulum
   Using the Time period Equation, we will get $l = 99.3cm \approx 1m$ Length of seconds pendulum

2. **Simple Pendulum of Length comparable to the radius of Earth(R).**
   Time Period of such a pendulum is given by,

$$T = 2\pi\sqrt{\frac{1}{g(\frac{1}{l}+\frac{1}{R})}}$$

2.1. When length of pendulum is very large, i.e.,

l >> R that is $l \to \infty$ so

$$T = 2\pi\sqrt{\frac{R}{g}}$$

as $\frac{1}{l} \to 0$ so T = 1.4hr = **84.6 minute**

TIME PERIOD OF SIMPLE PENDULUM IN ACCELERATING REFERENCE FRAME

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

where g_{eff} = Effective acceleration due to gravity in reference system = acceleration of the point of suspension with respect to ground

Take $g_{eff} = |\boldsymbol{g} - \boldsymbol{a}|$ where bold letters are vectors.

If forces are applied on mass then use pseudo-force concept.(Example is covered in the Lecture 6 on Problem Solving)

COMPOUND PENDULUM/ PHYSICAL PENDULUM

When a rigid body is suspended from an axis and made to oscillate about that then it is called compound pendulum. For these physical pendulum, we have

$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

where $I = moment\ of\ inertia\ of\ the\ rigid\ body\ about\ the\ point\ of\ suspension$

$I = I_{cm} + ml^2$ here taking k = gyration radius, so we write

$\boldsymbol{I = mk^2 + ml^2}$ as $I_{cm} = mk^2$

where $l = distance\ between\ point\ of\ suspension\ and\ center\ of\ mass$

So finally, we get,

$$T = 2\pi \sqrt{\frac{m(k^2+l^2)}{mgl}} = 2\pi \sqrt{\frac{k^2+l^2}{gl}}$$

T is minimum when $l = k$ and so

$$T_{min} = 2\pi \sqrt{\frac{2k}{g}}$$

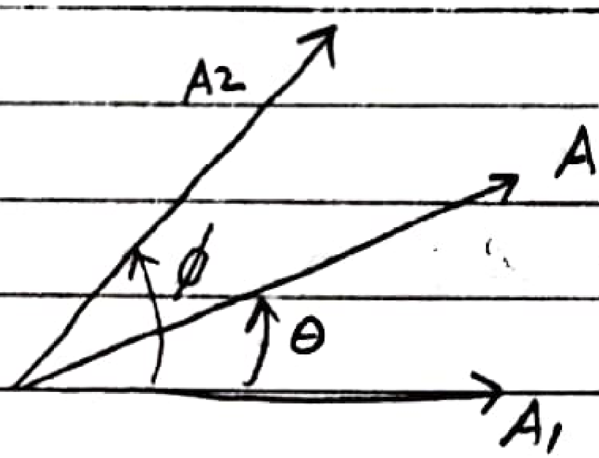
Examples are covered in the “related problems” section of Lecture 3

NOTES

1> SUPERPOSITION OF TWO SHM's

$x_1 = A_1 \sin\omega t$ $\qquad x_2 = A_2 \sin(\omega t + \phi)$

- same direction and same frequency



so, using Parallelogram Law,

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

and $\tan\theta = \dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

- same direction but different frequency

$x_1 = A_1 \sin\omega_1 t$ & $x_2 = A_2 \sin\omega_2 t$

So,

$$x = x_1 + x_2 = A_1\sin\omega_1 t + A_2\sin\omega_2 t$$

Not a SHM.

* DAMPED SIMPLE HARMONIC MOTION

if damping force is

$$F_d = -b\vec{v}.$$

so, our equation becomes

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

OR $$x(t) = Ae^{\frac{-bt}{2m}}\cos(\omega' t + \phi)$$

where,

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Total Mechanical Energy

$$E(t) = \frac{1}{2}kA^2e^{\frac{-bt}{m}}$$

* FORCED OSCILLATION (WITH DAMPING)

Taking force, $F = F_0\cos\omega_d t$

$\omega_d \equiv$ driven frequency

Equation becomes–

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos\omega_d t$$

so,

$$x = A\cos(\omega_d t + \phi)$$

$$A = \frac{F_0}{\{m^2(\omega^2-\omega_d^2) + \omega_d^2 b^2\}^{1/2}}$$

and $\tan\phi = \dfrac{-v_0}{\omega_d x_0}$ v_0 & x_0 are at time $t=0$

(a) <u>Small Damping</u> (b is small)

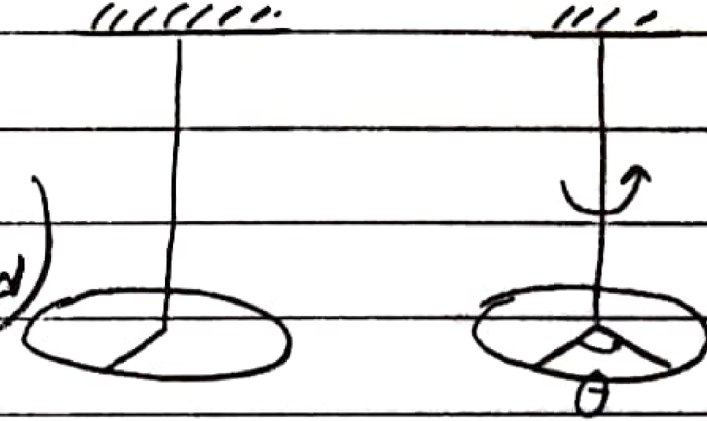
$$A = \frac{F_0}{m(\omega^2-\omega_d^2)}$$ i.e $\omega_d b$ is small.

(b) Driving Frequency 'ω' closer to 'ω_d'

$$A = \frac{F_0}{\omega_d b}$$

<u>Resonance occurs when</u> ($\omega = \omega_d$).

Torsional Pendulum



Torque in this case $\propto$ (angle the wire is rotated)

which gives

$\tau \propto \theta \Rightarrow \tau = -C\theta$ { C is torsional constant }

so, $\tau = I\alpha = -C\theta$

$\Rightarrow \alpha = -\left(\frac{C}{I}\right)\theta$ which gives $\omega^2 = \frac{C}{I}$

so, $T = 2\pi\sqrt{\frac{I}{C}}$ $I \equiv$ Moment of inertia about vertical axis.