* DAMPED SIMPLE HARMONIC MOTION

if damping force is

$$F_d = -b\vec{v}.$$

so, our equation becomes

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

OR

$$x(t) = A e^{\frac{-bt}{2m}} \cos(\omega' t + \phi)$$

where,

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Total Mechanical Energy

$$E(t) = \frac{1}{2} k A^2 e^{\frac{-bt}{m}}$$

* FORCED OSCILLATION (WITH DAMPING)

Taking force, $F = F_0 \cos \omega_d t$

$\omega_d \equiv$ driven frequency

Equation becomes-

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos \omega_d t$$

so,

$$x = A\cos(\omega_d t + \phi)$$

$$A = \frac{F_0}{\{m^2(\omega^2 - \omega_d^2) + \omega_d^2 b^2\}^{1/2}}$$

and $\tan\phi = \frac{-v_0}{\omega_d x_0}$ v_0 & x_0 are at time $t=0$

(a) <u>Small Damping</u> (b is small)

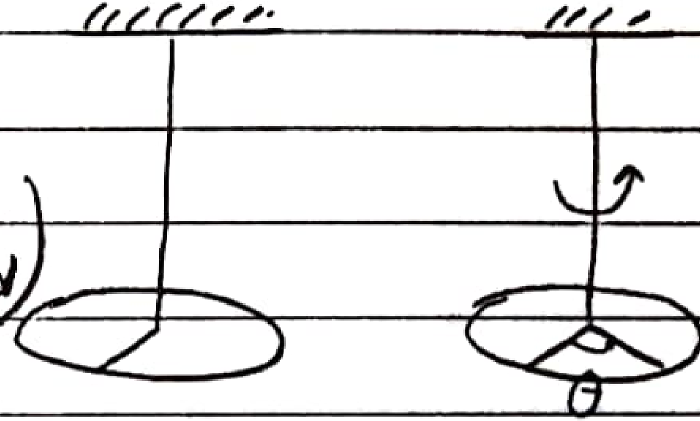
$A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$ i.e $\omega_d b$ is small.

(b) <u>Driving Frequency 'ω' closer to 'ω_d'</u>

$$A = \frac{F_0}{\omega_d b}$$

<u>Resonance occurs when ($\omega = \omega_d$).</u>

Torsional Pendulum



Torque in this case $\propto$ (angle the wire is rotated)

which gives

$\tau \propto \theta \Rightarrow \tau = -C\theta$ { $\because$ C is torsional constant }

so, $\tau = I\alpha = -C\theta$

$\Rightarrow \alpha = -\left(\frac{C}{I}\right)\theta$ which gives $\omega^2 = \frac{C}{I}$

so, $T = 2\pi\sqrt{\frac{I}{C}}$ $I \equiv$ Moment of inertia about vertical axis.