SIMPLE PENDULUM

If a heavy point-mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum.

Time Period of oscillation of simple pendulum of length l for small angular amplitude is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Note – We can take g = π^2 for making calculation simpler.

SPECIAL POINTS

1. **Seconds Pendulum** – T = 2s = Time Period of Seconds Pendulum
   Using the Time period Equation, we will get $l = 99.3cm \approx 1m$ Length of seconds pendulum

2. **Simple Pendulum of Length comparable to the radius of Earth(R).**
   Time Period of such a pendulum is given by,

$$T = 2\pi\sqrt{\frac{1}{g(\frac{1}{l}+\frac{1}{R})}}$$

2.1. When length of pendulum is very large, i.e.,

l >> R that is $l \to \infty$ so

$$T = 2\pi\sqrt{\frac{R}{g}}$$

as $\frac{1}{l} \to 0$ so T = 1.4hr = **84.6 minute**

TIME PERIOD OF SIMPLE PENDULUM IN ACCELERATING REFERENCE FRAME

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

where g_{eff} = Effective acceleration due to gravity in reference system = acceleration of the point of suspension with respect to ground

Take $g_{eff} = |\boldsymbol{g} - \boldsymbol{a}|$ where bold letters are vectors.

If forces are applied on mass then use pseudo-force concept.(Example is covered in the Lecture 6 on Problem Solving)

COMPOUND PENDULUM/ PHYSICAL PENDULUM

When a rigid body is suspended from an axis and made to oscillate about that then it is called compound pendulum. For these physical pendulum, we have

$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

where $I = moment\ of\ inertia\ of\ the\ rigid\ body\ about\ the\ point\ of\ suspension$

$I = I_{cm} + ml^2$ here taking k = gyration radius, so we write

$\boldsymbol{I = mk^2 + ml^2}$ as $I_{cm} = mk^2$

where $l = distance\ between\ point\ of\ suspension\ and\ center\ of\ mass$

So finally, we get,

$$T = 2\pi \sqrt{\frac{m(k^2+l^2)}{mgl}} = 2\pi \sqrt{\frac{k^2+l^2}{gl}}$$

T is minimum when $l = k$ and so

$$T_{min} = 2\pi \sqrt{\frac{2k}{g}}$$

Examples are covered in the “related problems” section of Lecture 3